

# Dynamic response of tubular joints with an annular void subjected to a harmonic axial load

A. Vaziri, H. Nayeb-Hashemi\*

*Department of Mechanical, Industrial and Manufacturing Engineering, Northeastern University, Boston, MA 02115-5000, USA*

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## Abstract

Dynamic responses of adhesively bonded tubular joints subjected to a harmonic axial load were evaluated with the use of a shear lag model. Adherents were assumed to be elastic and the adhesive was taken to be a viscoelastic material. Effects of tubular joint geometries, material properties and adhesive properties on the dynamic response of the system were investigated. The results showed that the system response was sensitive to the adhesive loss factor. The system response was little affected by the presence of a central annular void in the bond area for void size less than 40% of the overlap length. This was especially pronounced for joints using adhesives with a larger loss factor (viscous damping).

The distribution of shear stress amplitude in the joint area was obtained. The maximum shear stress was confined to the edge of the overlap for all applied loading frequencies. For the adhesive and adherents' properties and geometries investigated, the maximum shear stress amplitude in the joint area was little affected by the presence of a central annular void covering up to 40% of the overlap length. However, for a void larger than 40% of the overlap length, the maximum shear stress might increase or decrease with an increase in the void size. This was related to the applied loading frequency and the changes in the system resonance frequencies.

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## 1. Introduction

There has been a steady increase in the application of adhesives for joining structural components such as tubular structures. Traditional joining methods such as brazing, soldering, welding, etc. are often replaced in favor of adhesives. Joining components by adhesives not only reduces the manufacturing cost, it may also prolong the life of these components by reducing their vibrational response.

Despite many advantages, applications of adhesives are often proceeded with caution due to lack of reliable database concerning their long-term behavior under static and dynamic loadings in various environmental conditions. Furthermore, defects such as void, gap, improper surface preparation and curing can severely affect the bond quality and its static and dynamic

responses. There are many theoretical analyses addressing stress distribution and dynamic response of tubular joints. Earlier work on the stress distribution in tubular lap joint subjected to an axial load was done by Lubkin and Reissner [1]. An elastic stress distribution was obtained for tubes with thin circular cross-section subjected to an axial load. They modeled the adhesive layer as an infinite number of coil springs with the assumption that the adhesive was much softer than the adherents. Alwar and Nagaraja [2] solved the same problem by treating the adhesive as a viscoelastic material. The effect of a defect in the overlap area on the stress distribution in tubular joints subjected to axial and torsional loadings was obtained by Nayeb-Hashemi et al. [3]. The axial shear stress distribution was obtained with the use of a shear lag model and the torsional shear stress in the overlap area was obtained with the use of the analytical solution proposed by Adams and Peppiatt [4]. It was found that for certain tubular joint geometry and properties, it is possible for an annular void to have little effect on the bond strength. Chon [5] obtained the

\*Corresponding author. Tel.: +1-617-373-5515; fax: +1-617-373-2921.

*E-mail address:* hamid@coe.neu.edu (H. Nayeb-Hashemi).

stress field in tubular joints where adherents were composite materials. The stress concentration at and near the end of the joint was studied as a function of various joint properties and geometry. Terekhova and Skoryi [6] derived stress distributions in tubular lap joints under other loading conditions, such as external and internal pressures.

The dynamic response of tubular joints has been investigated by a number of investigators, who considered the adhesive both as elastic as well as a viscoelastic material [7–11]. The effects of various joint parameters on the system resonance frequencies and the system loss factor were investigated. The changes in the system resonance frequencies and modal shapes have also been used to predict the structural integrity of joint structures [12–15]. However, to best of our knowledge, there are no investigations on the effect of defects in the lap area on the dynamic response of the tubular joints. Sato and Ikegami [16] investigated the dynamic strength of adhesively bonded joints experimentally. The strength of bonded joints were measured at high strain rates using the clamped Hopkinson bar under tension and torsion stress waves. It was found that the dynamic strength of adhesive joints was greater than the static strength under tension and torsion.

The purpose of the work described in this paper is to understand the effects of tubular joint geometry and properties on the dynamic response when they are subjected to a harmonic axial load. The adhesive layer is assumed to be a viscoelastic material and the effect of its damping on the structural response is evaluated. In addition, effects of defects such as an annular void in the lap area on the dynamic response of tubular bonded joints are investigated.

## 2. Theoretical investigation

The shear stress in tubular bonded joints subjected to a harmonic axial load is obtained with the use of a shear lag model approach and with the assumption that adherents carry only axial load while the adhesive carries shear stress. This can be justified since the elastic moduli of most adherents are much greater than that of adhesives. The effect of the normal stress in the adhesive/adherents interface is also neglected. However, the shear stress can vary across the adhesive thickness. Furthermore, adherents are considered to be elastic materials and the adhesive to be a linear viscoelastic material. The shear modulus  $G_a$  of the adhesive is assumed to be  $G_a = G_0(1 + i\eta)$ , where  $G_0$  is the shear modulus and  $\eta$  is the adhesive loss factor. Effects of an annular void on the dynamic response and shear stress distribution are also investigated. Fig. 1 shows a schematic diagram of a tubular joint subjected to a harmonic axial load.

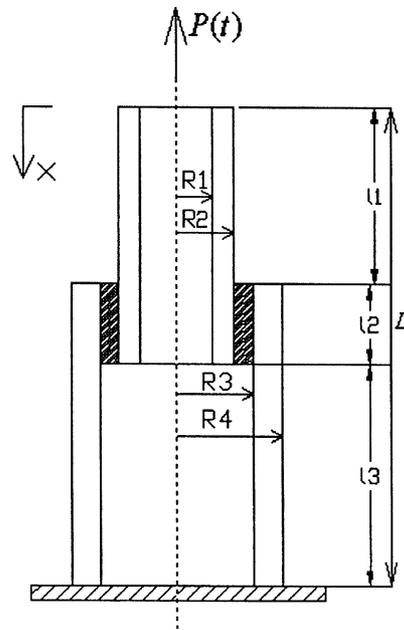


Fig. 1. Schematic diagram of a tubular bonded joint under a harmonic axial load.

Assuming the shear stresses in the inner and the outer surface of adherents 1 and 2 are  $(\tau_{rx})_1$  and  $(\tau_{rx})_2$ , and displacements of each adherent at the location  $x$  are  $\bar{u}_1$  and  $\bar{u}_2$ , respectively, equilibrium equations for an element in the overlap region for each adherent of length  $dx$  can be written as,

$$E_1 \pi (R_2^2 - R_1^2) \frac{\partial^2 \bar{u}_1}{\partial x^2} + (2\pi R_2) (\tau_{rx})_1 - \pi (R_2^2 - R_1^2) \rho_1 \bar{u}_1 = 0, \quad (1)$$

$$E_2 \pi (R_4^2 - R_3^2) \frac{\partial^2 \bar{u}_2}{\partial x^2} - (2\pi R_3) (\tau_{rx})_2 - \pi (R_4^2 - R_3^2) \rho_2 \bar{u}_2 = 0, \quad (2)$$

where  $E_1$  and  $E_2$  are elastic moduli and  $\rho_1$  and  $\rho_2$  are densities of adherents 1 and 2, and  $(R_1, R_2)$  and  $(R_3, R_4)$  are internal and external radiuses of adherent 1 and 2, respectively.

The equilibrium equation of an adhesive layer with a length of  $dx$  can be written as

$$\tau_a dr + r d\tau_a - r \rho_a \bar{u}_a dr = 0, \quad (3)$$

where  $\tau_a$  is the shear stress in the adhesive,  $\rho_a$  is the adhesive density and  $\bar{u}_a$  is the axial displacement of the adhesive layer. Assuming that the shear stress in the adhesive layer is  $\tau_a = G_a \partial \bar{u}_a / \partial r$ , Eq. (3) can be stated as

$$G_a \frac{\partial \bar{u}_a}{\partial r} + r \frac{\partial}{\partial r} \left( G_a \frac{\partial \bar{u}_a}{\partial r} \right) - r \rho_a \bar{u}_a = 0. \quad (4)$$

For a harmonic axial load, the adhesive and adherent displacements can be represented as  $\bar{u}_1(x, t) = u_1(x) e^{i\omega t}$ ,

$\bar{u}_2(x, t) = u_2(x)e^{i\omega t}$  and  $\bar{u}_a(r, t) = u_a(r)e^{i\omega t}$ . Substituting for the adhesive displacement, Eq. (4) can be presented as,

$$\frac{d^2 u_a}{dr^2} + \frac{1}{r} \frac{du_a}{dr} + \beta u_a = 0, \tag{5}$$

where

$$\beta = \sqrt{\frac{\rho_a \omega^2}{G_a}}. \tag{6}$$

Assuming displacements of adherents 1 and 2 are  $u_1$  and  $u_2$  at  $r = R_2$  and  $r = R_3$ , respectively, Eq. (5) can be solved to obtain the displacement field across adhesive thickness. The solution can be presented as,

$$u(r) = [Y_0(\beta R_3)J_0(\beta r) - J_0(\beta R_3)Y_0(\beta r)]\frac{u_1}{z} + [J_0(\beta R_2)Y_0(\beta r) - Y_0(\beta R_2)J_0(\beta r)]\frac{u_2}{z}, \tag{7}$$

where

$$z = J_0(\beta R_2)Y_0(\beta R_3) - J_0(\beta R_3)Y_0(\beta R_2). \tag{8}$$

Here  $J_0$  and  $Y_0$  are the Bessell function of order zero and the Bessell function of the second kind of order zero, respectively.

The shear stress at the adhesive/adherents interface can now be evaluated from Eq. (7) as,

$$(\tau_{rx})_1 = G_a \left. \frac{du_a}{dr} \right|_{r=R_2} = G_a[\alpha_1(R_2)u_1 + \alpha_2(R_2)u_2], \tag{9}$$

$$(\tau_{rx})_2 = G_a \left. \frac{du_a}{dr} \right|_{r=R_3} = G_a[\alpha_1(R_3)u_1 + \alpha_2(R_3)u_2], \tag{10}$$

where  $\alpha_1(r)$  and  $\alpha_2(r)$  are

$$\alpha_1(r) = [J_0(\beta R_3)Y_1(\beta r) - Y_0(\beta R_3)J_1(\beta r)]\frac{\beta}{z}, \tag{11}$$

$$\alpha_2(r) = [J_0(\beta R_2)Y_1(\beta r) - Y_0(\beta R_2)J_1(\beta r)]\frac{\beta}{z}, \tag{12}$$

$$\frac{du_a}{dr} = \alpha_1(r)u_1 + \alpha_2(r)u_2. \tag{13}$$

Here  $J_1$  and  $Y_1$  are the Bessell function of order one and the Bessell function of the second kind of order one, respectively.

Substituting for  $(\tau_{rx})_1$  and  $(\tau_{rx})_2$ , Eqs. (1) and (2) can be presented as,

$$E_1(R_2^2 - R_1^2) \frac{d^2 u_1}{dx^2} + 2R_2 G_a [\alpha_1(R_2)u_1 + \alpha_2(R_2)u_2] + (R_2^2 - R_1^2)\rho_1 \omega^2 u_1 = 0, \tag{14}$$

$$E_2(R_4^2 - R_3^2) \frac{d^2 u_2}{dx^2} + 2R_3 G_a [\alpha_1(R_3)u_1 + \alpha_2(R_3)u_2] + (R_4^2 - R_3^2)\rho_1 \omega^2 u_2 = 0. \tag{15}$$

The non-dimensionalized form of Eqs. (14) and (15) can be stated as,

$$\frac{d^2 u_1}{d\xi^2} + a_1 u_1 + a_2 u_2 = 0, \tag{16}$$

$$\frac{d^2 u_2}{d\xi^2} + a_3 u_1 + a_4 u_2 = 0, \tag{17}$$

where,  $\xi = x/L$  and

$$a_1 = \frac{2L^2 R_2 G_a \alpha_1(R_2)}{E_1(R_2^2 - R_1^2)} + \frac{\rho_1 \omega^2 L^2}{E_1}, \tag{18}$$

$$a_2 = \frac{2L^2 R_2 G_a \alpha_2(R_2)}{E_1(R_2^2 - R_1^2)}, \tag{19}$$

$$a_3 = -\frac{2L^2 R_3 G_a \alpha_1(R_3)}{E_2(R_4^2 - R_3^2)}, \tag{20}$$

$$a_4 = -\frac{2L^2 G_a R_3 \alpha_2(R_3)}{E_2(R_4^2 - R_3^2)} + \frac{\rho_2 \omega^2 L^2}{E_2}, \tag{21}$$

where  $L$  is the over-all joint length, Fig. 1.

Eqs. (16) and (17) are valid in the overlap region ( $l_2$ ). Equilibrium equations for the first and the third regions ( $l_1, l_3$ ) can be written as

$$\frac{d^2 u_1}{d\xi^2} + a_5 u_1 = 0, \tag{22}$$

$$\frac{d^2 u_2}{d\xi^2} + a_6 u_2 = 0, \tag{23}$$

where

$$a_5 = \frac{\rho_1 \omega^2 L^2}{E_1} \tag{24}$$

and

$$a_6 = \frac{\rho_2 \omega^2 L^2}{E_2}. \tag{25}$$

Eqs. (16) and (17) can be simultaneously solved to obtain the displacement field in the overlap region. This can be presented as,

$$u_1 = \sum_{j=1}^4 A_{2j} e^{S_{2j}\xi}, \tag{26}$$

$$u_2 = \sum_{j=1}^4 t_j A_{2j} e^{S_{2j}\xi}, \tag{27}$$

where  $S_{2j}$  ( $j = 1$  to  $4$ ) are the roots of  $S^4 + (a_1 + a_4)S^2 + (a_1 a_4 - a_2 a_3) = 0$  \tag{28}

and

$$t_j = -\frac{S_j^2 + a_1}{a_2}. \tag{29}$$

Similarly, displacement field in regions (1) and (3) can be presented as

$$u_1 = \sum_{j=1}^2 A_{1j} e^{S_{1j}\xi}, \text{ where}$$

$$S_{11} = i\sqrt{a_5} \text{ and } S_{12} = -i\sqrt{a_5} \tag{30}$$

and

$$u_2 = \sum_{j=1}^2 A_{3j} e^{S_{3j}\xi}, \text{ where}$$

$$S_{31} = i\sqrt{a_6} \text{ and } S_{32} = -i\sqrt{a_6}. \tag{31}$$

Boundary and continuity in the various regions of the overlap are

(i) at  $\xi = 0$

$$\frac{du_1}{d\xi} = \frac{PL}{E_1\pi(R_2^2 - R_1^2)}, \tag{32}$$

(ii) at  $\xi = l_1/L$

$$u_1|_{\text{region 1}} = u_1|_{\text{region 2}}, \tag{33}$$

$$\left. \frac{du_1}{d\xi} \right|_{\text{region 1}} = \left. \frac{du_1}{d\xi} \right|_{\text{region 2}}, \tag{34}$$

$$\left. \frac{du_2}{d\xi} \right|_{\text{region 2}} = 0, \tag{35}$$

(iii) at  $\xi = (l_1 + l_2)/L$

$$\left. \frac{du_1}{d\xi} \right|_{\text{region 2}} = 0, \tag{36}$$

$$u_2|_{\text{region 2}} = u_2|_{\text{region 3}}, \tag{37}$$

$$\left. \frac{du_2}{d\xi} \right|_{\text{region 2}} = \left. \frac{du_2}{d\xi} \right|_{\text{region 3}}, \tag{38}$$

(iv) at  $\xi = 1$

$$u_2|_{\text{region 3}} = 0. \tag{39}$$

Similar equations are also developed for tubular joints containing an annular void in the overlap area by dividing the overlap to three regions [17]. Here we will present the results without stating the corresponding equations.

### 3. Results and discussion

The results provided here are for the tubular bonded joint configuration shown in Fig. 1. The adherents and adhesive were taken to be 6061-T6 aluminum with elastic modulus of 69 GPa and density of 2710 kg/m<sup>3</sup> and a shell epoxy with the real part of the shear modulus of 791 MPa and the density of 1200 kg/m<sup>3</sup>, respectively. The upper adherent was considered to be a solid cylinder with the radius of 17.8 mm and the lower adherent was assumed to be a tube with the inner and

outer radii of 19.0 and 25.4 mm, respectively. The lengths of the upper and lower adherents were 250 mm. The overlap length was assumed to be 50 mm in all analyses except when it was desired to understand the effect of the overlap length on the dynamic response of the tubular joint. Adhesive was assumed to behave as a viscoelastic material with various loss factors,  $\eta$ . The effect of adherent and adhesive material properties on the system response was also investigated by keeping the elastic modulus of the upper adherent constant ( $E_1 = 69$  GPa), while changing the elastic modulus and shear modulus of the lower adherent and adhesive respectively. MATLAB based codes were written to analyze displacements and relating shear stresses.

Fig. 2 shows the frequency response of the bonded tubular joint at the point of applied load. The frequency response was obtained for adhesives with loss factor,  $\eta$ , ranging from 0 to 1. The results indicate that the adhesive damping affects the amplitude of the vibration and has relatively little effect on the lower resonance frequencies. However, adhesive damping apparently shifts the higher resonance frequencies. The results indicate that, as expected, the amplitude of the system reduces with an increase in the adhesive loss factor.

Fig. 3 shows the effects of adhesive and adherent elastic modulus on the first resonance frequency of tubular joints. The results indicate that the first resonance frequency of tubular joints is higher for joints with a higher  $E_2/E_1$ . The results also show that the first natural frequency of tubular joints increases rapidly when using adhesive with a higher shear modulus. However, beyond a certain shear modulus the first resonance frequency becomes less sensitive to the adhesive shear modulus. These results could be justified considering the shear stress distribution in the adhesive/adherent interface and its variation with the adhesive shear modulus.

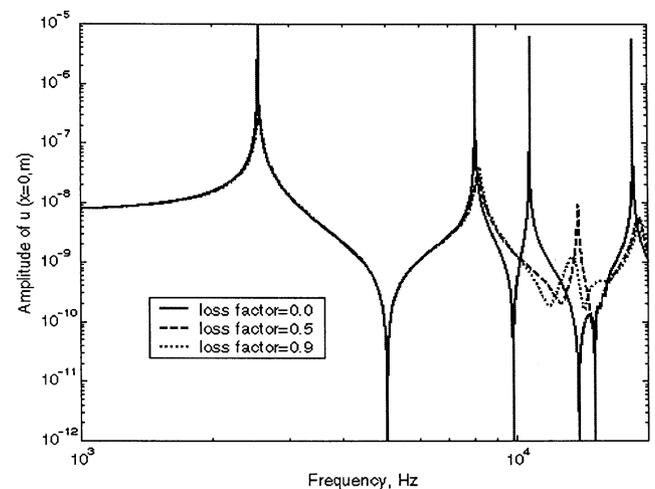


Fig. 2. Effect of adhesive loss factor  $\eta$  of 0, 0.5 and 0.9 on the frequency response of a tubular joint.

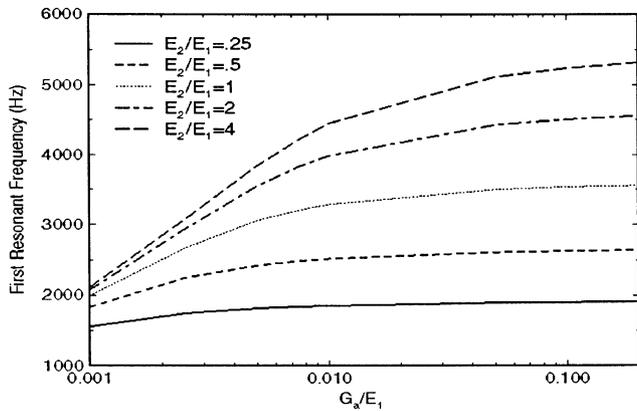


Fig. 3. Effect of adhesive/adherents elastic modulus on the first natural frequency of tubular joints.

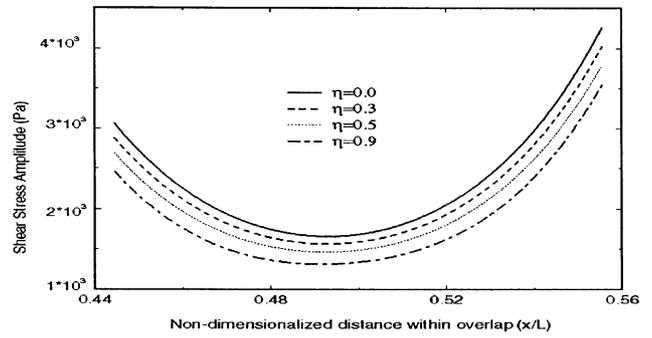


Fig. 5. Distribution of the shear stress amplitude at the adhesive/adherent 1 interface for joints with various adhesive loss factors,  $\eta$ . Tubular joints were subjected to an axial load of 1 N at frequency of 2000 Hz.

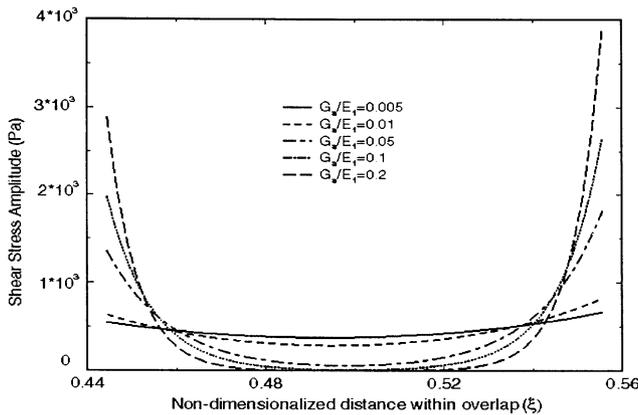


Fig. 4. Distribution of the shear stress amplitude at the adhesive/adherent 1 interface for joints with various  $G_a/E_1$  and adhesive loss factor,  $\eta = 0$ . Tubular joints were subjected to an axial load of 1 N at frequency of 2000 Hz.

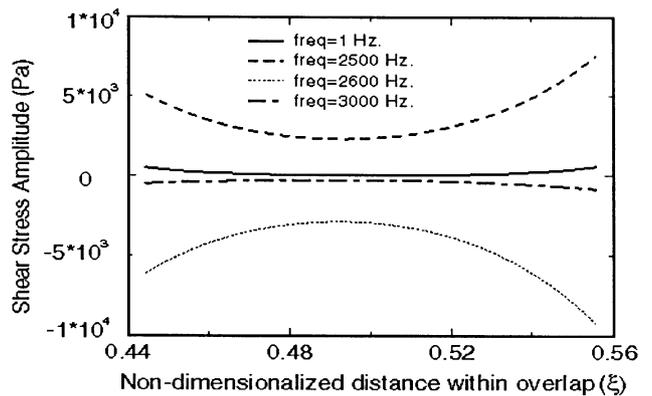


Fig. 6. Distribution of the shear stress amplitude at the adhesive/adherent 1 interface for a joint subjected to an axial load of 1 N at various frequencies. Adhesive was assumed to be elastic,  $\eta = 0$ .

Fig. 4 shows the distribution of the shear stress amplitude at the adhesive/adherent 1 interface for tubular joints with  $E_2/E_1 = 1$  and subjected to a harmonic load with amplitude of 1 N at a frequency of 2000 Hz. This frequency is lower than the system's first resonance frequency and thus provides a better resolution on the effect of adhesive mechanical properties on the shear stress distribution in the overlap area. The results show that the shear stress is distributed fairly uniformly in the overlap for joints with  $G_a/E_1 = 0.005$ . However, for a higher  $G_a/E_1$ , the shear stress distribution is not uniform and the maximum shear stress amplitude is located at the edges of the overlap. Furthermore, the maximum shear stress amplitude is smaller for joints with higher adhesive damping, Fig. 5. The shear stress amplitude distribution is a function of the applied loading frequency and it increases as the loading frequency approaches the system resonance frequency, Fig. 6. The shear stress is in phase with the loading frequency when the loading frequency is smaller

than the first natural frequency, and it is out of phase when the frequency is greater than the first natural frequency. Furthermore, the results indicate that for some joint geometries and properties, a portion of the overlap may not contribute to the over-all resistance of the tubular joint to an axial load since it has almost zero shear stress and may be considered to be a dead area. The length of this dead zone depends on the relative elastic modulus of adhesive to adherents. In contrast, the natural frequencies of the tubular joint depend on the length of the dead zone. Fig. 7 shows the natural frequency of the tubular joint as a function of overlap length. Results indicate that the first natural frequency of the system initially increases with an increase of the overlap length. However, further increase in the overlap length reduces the first natural frequency of the system. These results could be justified considering the dead region in the overlap contribute very little to the over all system stiffness, but it increases the total mass of the system.

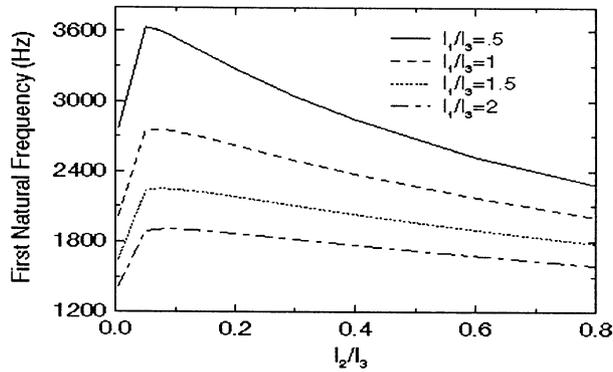


Fig. 7. Effect of the overlap length on the first natural frequency of tubular joints.

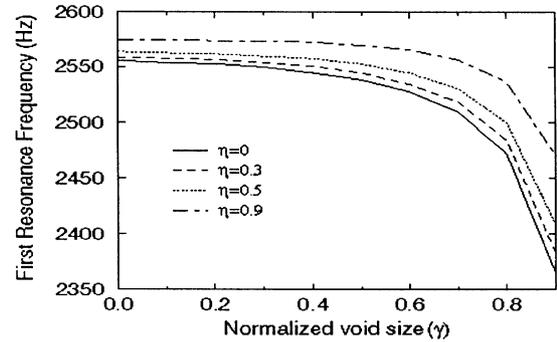


Fig. 9. Effect of void size on the first resonant frequency of a tubular joint with various adhesive loss factors.

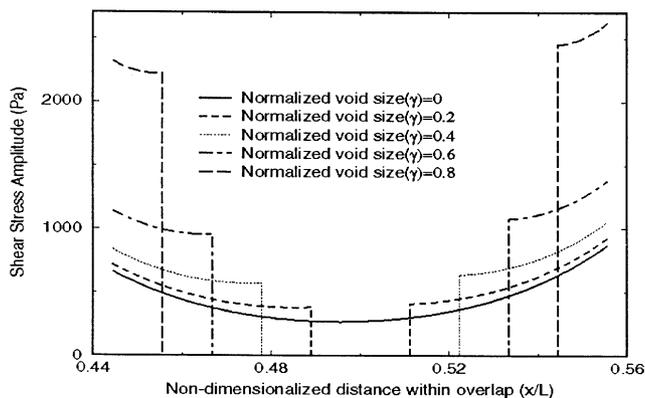


Fig. 8. Shear stress distribution within the overlap section for a tubular joint with the adhesive loss factor,  $\eta = 0$  and with various central void sizes, subjected to an axial load of 1 N at frequency = 2000 Hz.

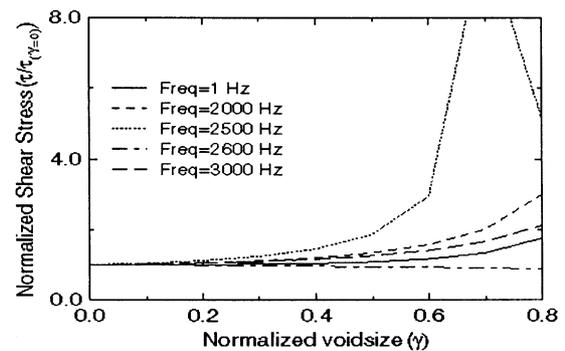


Fig. 10. Maximum shear stress versus normalized void size  $\gamma$ , for a tubular joint subjected to a 1 N harmonic axial force at various frequencies. Adhesive was assumed to be elastic,  $\eta = 0$ .

The effect of an annular central void on the shear stress amplitude distribution and the first natural frequency of the joint is shown in Figs. 8 and 9, respectively. Here void size is defined as  $\gamma$  and is equal to void size/overlap length. Fig. 9 shows that there is relatively little change in the first natural frequency of the joint with a central void size,  $\gamma$ , of less than 40%. The first natural frequency drastically reduces for joints with a larger void size. The reduction in the natural frequency with a presence of a void may be beneficial or detrimental to the joint strength depending on the applied loading frequency. For joints subjected to a harmonic load at a lower frequency than the natural frequency of the joint without a void, a void may bring the system's natural frequency closer to the applied loading frequency, thus increasing the maximum shear stress amplitude in the joint, Figs. 9–11. However, for joints subjected to a loading frequency greater than the system's first natural frequency, a void will further depart the system from the resonance frequency, thus developing smaller shear stress at the edge of the overlap, Figs. 9–12.

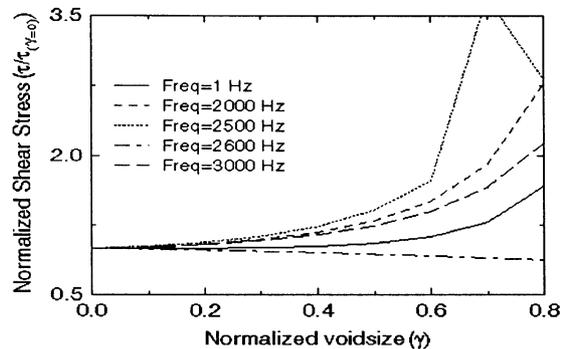


Fig. 11. Maximum shear stress versus normalized void size  $\gamma$ , for a tubular joint subjected to a 1 N harmonic axial force at various frequencies. Adhesive was assumed to be viscoelastic,  $\eta = 0.5$ .

#### 4. Conclusions

Dynamic response of adhesively bonded tubular joints subjected to a harmonic axial load is obtained as a function of adherents' mechanical properties and geometry, as well as the adhesive mechanical and damping properties. In addition, effects of defects such

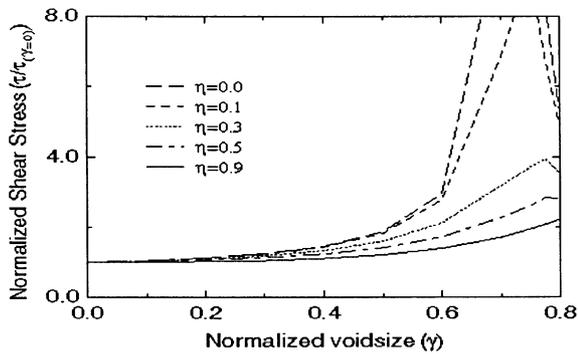


Fig. 12. Maximum shear stress versus normalized void size  $\gamma$ , for a tubular joint subjected to a 1 N harmonic force at the frequency of 2500 Hz. Adhesive was assumed to be viscoelastic with various loss factors  $\eta$ .

as void in the overlap area, on the system dynamic response are investigated. It was found,

Adhesive loss factor,  $\eta$ , has little effect on the lower resonance frequencies, however, it apparently affects higher resonance frequencies.

The first resonance frequency of the system increases rapidly with an increase in  $G_a/E_1$  (adhesive/adherent elastic modulus). However, beyond a certain value, this increase is less pronounced.

Shear stress distribution in the overlap is obtained for a tubular joint subjected to an axial harmonic load at several loading frequencies. The maximum shear stress is confined to the overlap region. The maximum shear stress reduces for joints with a larger adhesive loss factor. Furthermore, for joints with  $G_a/E_1 > 0.05$ , the middle section of the overlap has almost zero shear stress. This region is termed dead zone. The first natural frequency of the system initially increases with an increase in the overlap length. However, further increase reduces the system resonance frequency. This was related to the length of the dead zone.

For adhesive and adherent properties and geometry studied, the first resonance frequency is little affected with the presence of an annular central void in the overlap area for a void size of less than 40% of the overlap length. However, further increase in the void size drastically reduces the system resonance frequency.

This change in the first natural frequency may prolong the life of joint or reduce it. This depends on the applied loading frequency.

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