Dynamic response of adhesively bonded single-lap joints with a void subjected to harmonic peeling loads

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Abstract: The only viable method to join some components is by using adhesives. These components are often subjected to dynamic loading, which may cause initiation and propagation of failure in the joint. In order to ensure the reliability of these structures, their dynamic response and its variation with the presence of defects in the bonded area must be understood.

The dynamic response of a single-lap joint subjected to an out-of-plane harmonic force is evaluated. The bonded joint is modelled as a Euler–Bernoulli beam joined with an adhesive and constrained at one end and subjected to a harmonic force at the free end. The results show that the system response is not sensitive to a range of adhesive loss factor of 0–1. Furthermore, the system response is little affected by the presence of void in the bond area. The system response seems to be more sensitive to the void location than to its size. Peel and shear stresses in the bond area are obtained and found to be confined to the edge of the overlap. For the adhesive and adherent properties and geometries investigated the maximum peel and shear stresses in the bond area are little affected by the presence of a central void, covering up to 60 per cent of the overlap length for all applied loading frequencies. However, when the frequency of the applied load is close to the natural frequencies of the structure, a void may increase or decrease both the maximum peel and shear stresses.

Keywords: lap joint with void, harmonic vibration, natural frequency, material damping, adherent properties

NOTATION

A_i	cross-sectional area of the ith adherent
E_a	modulus of elasticity of the adhesive
E_i	modulus of elasticity of the ith adherent
$F(t)$	applied force
G	complex shear modulus of the adhesive
G_0	real part of the shear modulus of the adhesive
h_i	thickness of the ith adherent
I_i	moment of inertia of the ith adherent
k	stiffness per unit length of the adhesive
l_i	length of the ith section
L	overall length of the bonded joint
t	adhesive thickness
w	width of the beams
y_i	transverse displacement of the ith section
η	adhesive loss factor
ρ_i	density of the ith adherent
τ	shear stress
ω	peeling load frequency

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1 INTRODUCTION

Joining components by using adhesives is becoming more popular with the development of adhesives with high adhesion properties. Adhesively bonded joints offer many advantages compared with other methods of joining such as mechanical fastening, welding, brazing, etc. The ease of manufacturing, surface appearances, stress distribution in the bonded area, cost of manufacturing and capabilities of joining dissimilar materials are some of the advantages of using adhesives. Despite these benefits, the application of adhesives in joining critical components has proceeded with caution.

The properties of the bonded joint can be severely compromised by the defects produced during manufacturing. Defects can be in a form of disbond due to improper surface preparation of adherents, voids and deteriorated adhesive properties due to improper manufacturing processes. In the past few years, there have been a number of efforts to understand the effect of these defects on the bond strength. In addition, effort has been made to identify suitable non-destructive evaluation techniques for bond strength evaluation.

Nayeb-Hashemi and co-workers [**1–4**] studied the effects of a void on single-lap joints subjected to axial tensile force, tubular joints subjected to combined axial and torsional loadings, and single-lap joints subjected to a peeling load. It was found that the maximum stress occurs at the edge of the overlap. For certain bonded joint geometry and property, bond strength is little affected by the presence of a central void, covering up to 70 per cent of the overlap length. Furthermore, several ultrasonic techniques were used to evaluate the bond strength. It was found that the standard acousto-ultrasonic technique [**5**, **6**] provides no correlation between acousto-ultrasonic parameters and the specimen's bond strength. This was due to incorporation of all bond area in the bond strength prediction. New acousto-ultrasonic parameters, which take into account the quality of the adhesive and its location with respect to the overlap, were proposed [**1**]. It was found that these acousto-ultrasonic parameters provide a better prediction of the bond strength. The acoustic emission (AE) activities of bonded specimens were also monitored. The results showed that a majority of activities are initiated from the edge of the overlap. AE parameters such as AE energy and events were related to the bond strength.

Tong [**7**] developed a simple solution procedure for predicting the strength of adhesively bonded single-lap joints with non-linear adhesive properties. It was shown that neglecting the terms related to the transverse shear forces could yield an overestimation of both peel and shear strain energy rates for the single-lap joints with relatively stiff adhesive. Her [**8**] used a simplified one-dimensional model based on the classical elasticity theory to analyse the stresses in single-lap and double-lap joints. The analytical solution was compared with the numerical solution determined by the two-dimensional finite element method.

There are a number of papers on the effects of large displacement on the stress distribution in adhesively bonded joints, considering the adhesive as elastic, viscoelastic and viscoplastic material [**9–11**]. These analyses have resulted in identification of critical zones for joint failure. However, these analyses did not consider the effects of defects and dynamic loadings on the stress distribution and critical zones. Olia and Rossettos [**12**] derived the stress distribution in adhesively bonded joints with a gap subjected to bending. The results showed steep edge gradients for peel and shear stresses. The peel stresses always occurred at the extreme end of the overlap. It was found that a void had little effect on the peak stresses unless it was sufficiently close to an end.

Several theories have been proposed for predicting the failure of joints under a triaxial stress field [**13–17**]. Lee and Kong [**13**] and Imanka and co-workers [**14**, **15**] investigated fracture, fatigue failure and yield behaviour of adhesively bonded joints for two types of adhesively bonded joint. The results showed that both yield and fracture criteria depended on the stress triaxiality and that the fracture mechanism of the homogeneous adhesive was different from that of the heterogeneous one.

Dynamic responses of the bonded joints have been investigated by a number of researchers [**18–23**]. These investigations have been aimed towards finding the natural frequencies and corresponding mode shapes and the effects of bonded joint properties and geometries on the natural frequencies and mode shapes. Sato and Ikegami [**24**] investigated the dynamic strength of adhesively bonded joints using the clamped Hopkinson bar method. It was found that the dynamic strength of the bonded joints was greater than the static strength under tensile and shear loads.

Despite this body of literature, there is little information regarding the relation between vibration response and bond quality. The knowledge of the variation of natural frequencies and damping behaviour of the bonded joint with the bond geometry, adhesive properties and defects can highlight the existence of such a correlation. The purpose of this paper is to study the dynamic response of a single-lap joint with a void subjected to harmonic peeling load and its variation with the bond geometry and adhesive properties.

2 THEORETICAL INVESTIGATION

A schematic diagram of a single-lap joint subjected to a harmonic peeling load, F, is shown in Fig. 1. It is assumed that the bond line consists of a region where the adhesive is missing and creating a void in the bond area. The stress distribution in the bonded area is obtained by modelling adherents as Euler–Bernoulli beams supported on a viscoelastic foundation which resists both peeling and shear deformations. The bonded joint is divided into eight regions and governing equations for the transverse displacement, y_i, of each region are developed. The peel stress is assumed to be proportional to $k(y_i - y_j)$, where y_i and y_j are displacements of adherents right at the top and bottom of the adhesive layer in a particular region and k is the stiffness of the elastic foundation and is assumed to be $k = (E_a w/t)$, where E_a is the elastic modulus of the

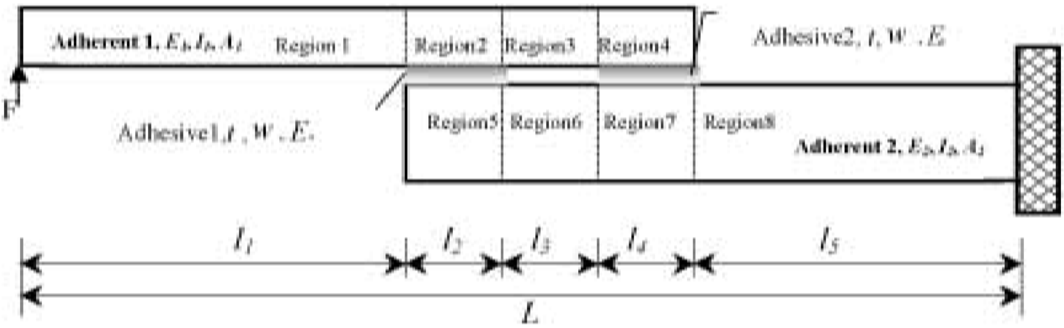

Fig. 1 Schematic model for a single-lap joint with void

adhesive layer, w is the width of the overlap and t is the adhesive thickness.

The shear stress distribution in the bonded area is given as [2]

$$\tau = \frac{G}{2t}\left(h_1 \frac{\mathrm{d}y_i}{\mathrm{d}x} + h_2 \frac{\mathrm{d}y_j}{\mathrm{d}x}\right) \tag{1}$$

where G is the adhesive shear modulus and h_1 and h_2 are the adherents' thicknesses. The adhesive is considered to be viscoelastic and its shear modulus is assumed to be

$$G = G_0(1 + i\eta) \tag{2}$$

where η is the adhesive loss factor.

Considering a free body diagram for region 2 (Fig. 2), the equilibrium equations are

$$\frac{\partial V}{\partial x} + \ddot{y}_2(\rho A)_1 + k(y_2 - y_5) = 0 \tag{3}$$

$$V - \frac{\partial M}{\partial x} + \tau w \frac{h_1}{2} = 0 \tag{4}$$

where V and M are shear force and bending moment respectively. Similarly, the equation of motion for the fifth region can be presented as

$$\frac{\partial V}{\partial x} + (\rho A)_2\, \ddot{y}_5 + k(y_5 - y_2) = 0 \tag{5}$$

$$V - \frac{\partial M}{\partial x} + \tau w \frac{h_2}{2} = 0 \tag{6}$$

Considering the relation between transverse displacement and bending moment from the beam theory:

$$M = (EI)\frac{\partial^2 y}{\partial x^2} \tag{7}$$

Equations (3), (4) and (7) can be combined to give

$$(EI)_1 \frac{\partial^4 y_2}{\partial x^4} + (\rho A)_1 \ddot{y}_2 + k(y_2 - y_5) + w\frac{h_1}{2}\frac{\partial \tau}{\partial x} = 0 \tag{8}$$

Similarly, equations (5), (6) and (7) can be simplified to

$$(EI)_2 \frac{\partial^4 y_5}{\partial x^4} + (\rho A)_2 \ddot{y}_5 + k(y_5 - y_2) - w\frac{h_2}{2}\frac{\partial \tau}{\partial x} = 0 \tag{9}$$

Similar equations are also developed for other regions of the joint [25]. For a bonded joint subjected to a harmonic

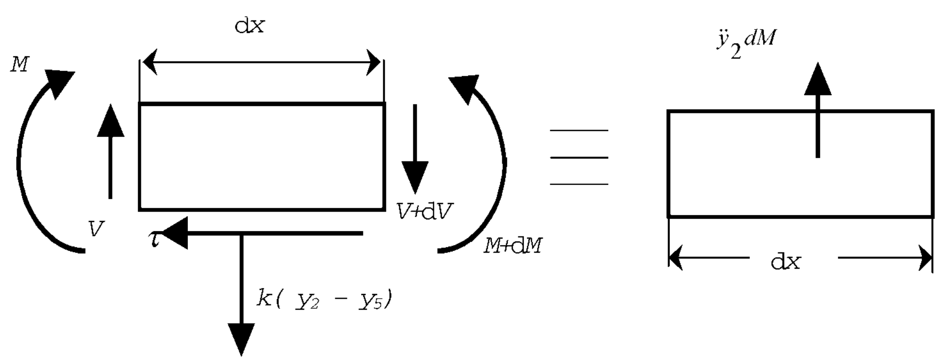

Fig. 2 Free body diagram of an element in region 2

peeling load of $Fe^{i\omega t}$ at $x = 0$, the displacement functions for each region will be in the form of

$$y_i(x, t) = y_i(x)e^{i\omega t} \qquad \text{where } i = 1, 2, \ldots, 8 \tag{10}$$

The governing equations for displacement of the bonded joint are non-dimensionalized, using the following non-dimensional parameters:

$$\beta_i = \frac{G_0}{E_i} \tag{11}$$

$$a_1 = \frac{(\rho A)_1 \omega^2 L^4}{(EI)_1} \tag{12}$$

$$a_2 = \frac{kL^4}{(EI)_1} \tag{13}$$

$$a_3 = \beta_1 \frac{w h_1^2 L^2}{4tI_1}(1 + i\eta) \tag{14}$$

$$a_4 = \frac{(\rho A)_2 \omega^2 L^4}{(EI)_2} \tag{15}$$

$$a_5 = \beta_1 \frac{w h_1 h_2 L^2}{4tI_1}(1 + i\eta) \tag{16}$$

$$a_6 = \beta_2 \frac{w h_1 h_2 L^2}{4tI_2}(1 + i\eta) \tag{17}$$

$$a_7 = \beta_2 \frac{w h_2^2 L^2}{4tI_2}(1 + i\eta) \tag{18}$$

$$a_8 = \frac{kL^4}{(EI)_2} \tag{19}$$

$$\bar{y}_i = \frac{y_i}{h_1} \tag{20}$$

$$\zeta = \frac{x}{L}, \qquad \zeta_1 = \frac{l_1}{L}, \qquad \zeta_2 = \frac{l_1 + l_2}{L}$$
$$\zeta_3 = \frac{l_1 + l_2 + l_3}{L} \qquad \text{and} \qquad \zeta_4 = \frac{l_1 + l_2 + l_3 + l_4}{L} \tag{21}$$

where $L = l_1 + l_2 + l_3 + l_4 + l_5$. The non-dimensionalized forms of equations (8) and (9) are

$$\frac{\partial^4 \bar{y}_2}{\partial \zeta^4} - a_1 \bar{y}_2 + a_2(\bar{y}_2 - \bar{y}_5) - a_3 \frac{\partial^2 \bar{y}_2}{\partial \zeta^2} - a_5 \frac{\partial^2 \bar{y}_5}{\partial \zeta^2} = 0 \tag{22}$$

$$\frac{\partial^4 \bar{y}_5}{\partial \zeta^4} - a_4 \bar{y}_5 + a_8(\bar{y}_5 - \bar{y}_2) - a_6 \frac{\partial^2 \bar{y}_2}{\partial \zeta^2} - a_7 \frac{\partial^2 \bar{y}_5}{\partial \zeta^2} = 0 \tag{23}$$

Similar non-dimensionalized equations were also developed for other regions and solutions were obtained by identifying matching boundary conditions between regions. For typical governing equations of displacements [equations (22) and (23)], the solution for the displacement fields is in the form of

$$\bar{y}_2 = \sum_{j=1}^{8} A_{2j} e^{S_{2j}\zeta} \qquad \text{and} \qquad \bar{y}_5 = \sum_{j=1}^{8} t_j A_{2j} e^{S_{2j}\zeta} \tag{24}$$

where S_{2j} are roots of the characteristic equation and t_j ($j = 1$ to 8) is given by

$$t_j = \frac{S_{2j}^4 - a_3 S_{2j}^2 + (a_2 - a_1)}{a_5 S_{2j}^2 + a_2} \tag{25}$$

3 RESULTS AND DISCUSSIONS

The results provided here are for the bonded joint configuration shown in Fig. 1. The adherents are joined together using Hysol EA 9689 epoxy film of 0.13 mm thickness with an elastic modulus of $E_a = 2.2$ GPa. The adherents are 6061-T6 aluminium with an elastic modulus of 69 GPa and a total overlap length of 25.4 mm with a width of 25.4 mm. The thickness of the upper and lower adherents are 3.17 and 12.6 mm respectively. MATLAB-based codes were written to analyse displacements and relating shear and peeling stresses. The location of void in the overlap and its size were systematically changed and their effects on the peel and shear stresses and natural frequencies of the bonded joint were investigated. Void size with respect to the overlap length is defined as

$$\xi = \frac{\text{void size}}{\text{overlap length}}$$

Figure 3 shows the frequency response of the bonded joint at the point of applied load. The frequency response was obtained for adhesives with loss factor η, ranging from 0 to 1. The results show that the adhesive damping property has very little effect on the frequencies where peak responses occur. The results also show that the first natural frequency corresponds to that of adherent 1, clamped at one end. This could be justified considering the thickness of the second adherent and realizing the stress distribution in the bonded area.

The effect of a void on the first natural frequency of the bonded joint was investigated. The change in the first natural frequency of the system was defined as γ (the first natural frequency of the system with a void/the first natural frequency of the system without a void). The results show that the void location apparently has a greater effect on natural frequencies that the void size

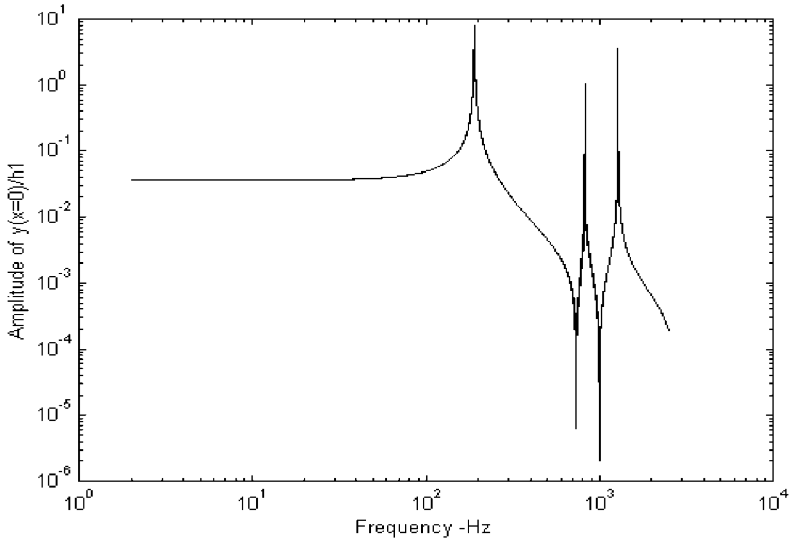

Fig. 3 Effect of the adhesive loss factor η of 0, 0.5 and 0.9 on the frequency response of the bonded joint

itself. For voids located at a distance greater than 20 per cent of the overlap length from one edge (l_2, Fig. 1), void size has little effect on the natural frequencies (Figs 4 to 6). The results indicate that natural frequencies are more sensitive to the void size for bonded joints with a smaller E_a/E_1 (Figs 4 to 6).

The distribution of peel and shear stresses in the overlap area for a bonded joint with a central void subjected to a harmonic peeling load, at frequencies of 1 and 185 Hz (the first natural frequency ~188 Hz), are shown in Figs 7 to 10. The results show that the maximum peel and shear stresses are located near the edge of the overlap. The results also indicate that the maximum peel and shear stresses are not affected significantly with the existence of a central void size, covering up to 60 per cent of the overlap area, for all applied loading frequency ranges (Figs 11 and 12). The maximum peel and shear stresses are apparently more sensitive to the void size when the frequency of the applied load is close to the bonded joint natural frequencies (Figs 11 to 14).

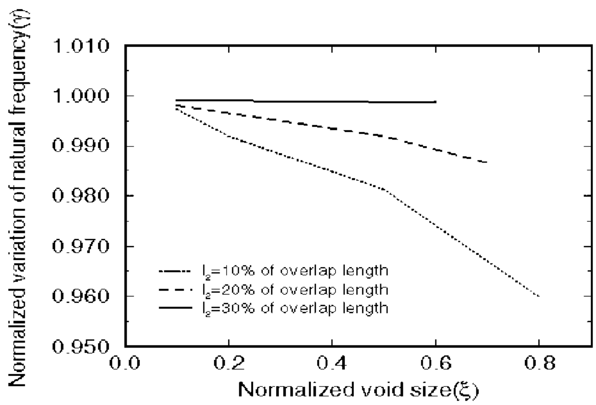

Fig. 5 Effect of the void size on the first natural frequency of the bonded joint with $E_a/E_1 = 0.01$

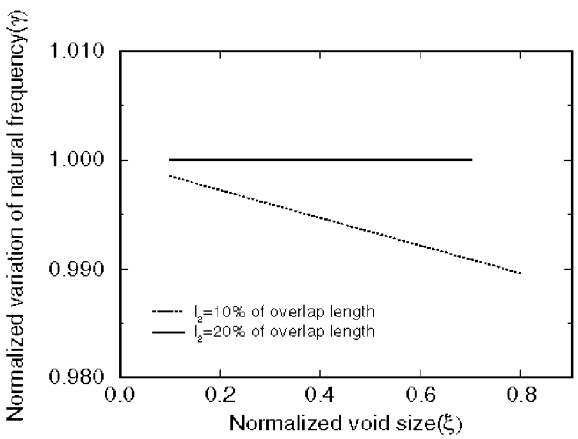

Fig. 4 Effect of the void size on the first natural frequency of the bonded joint with $E_a/E_1 = 0.1$

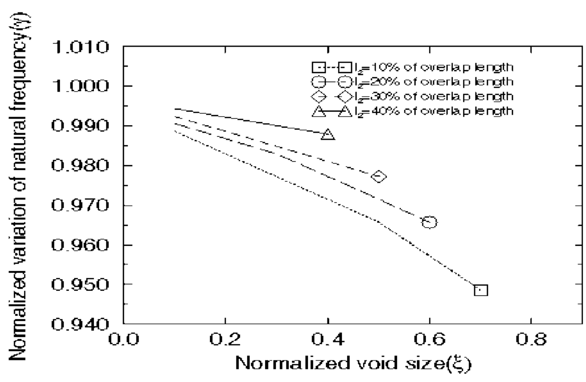

Fig. 6 Effect of the void size on the first natural frequency of the bonded joint with $E_a/E_1 = 0.001$

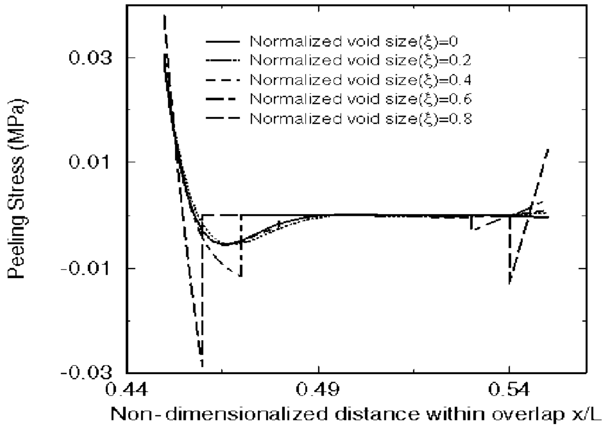

Fig. 7 Peel stress distribution in the bonded joint with various central void sizes. The joint was subjected to a harmonic peeling load of 1 N with the frequency of 1 Hz

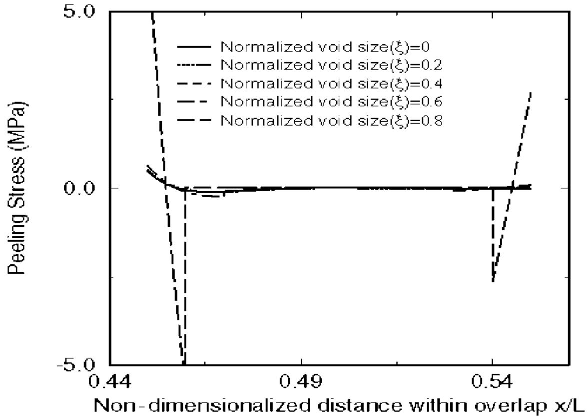

Fig. 8 Peel stress distribution in the bonded joint with various central void sizes. The joint was subjected to a harmonic peeling load of 1 N with the frequency of 185 Hz

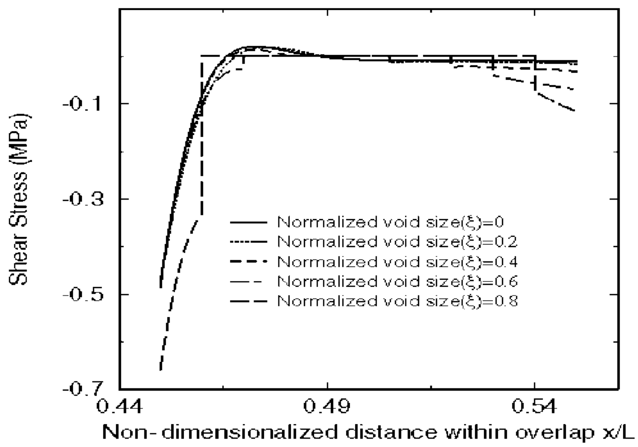

Fig. 9 Shear stress distribution in the bonded joint with various central void sizes. The joint was subjected to a harmonic peeling load of 1 N with the frequency of 1 Hz

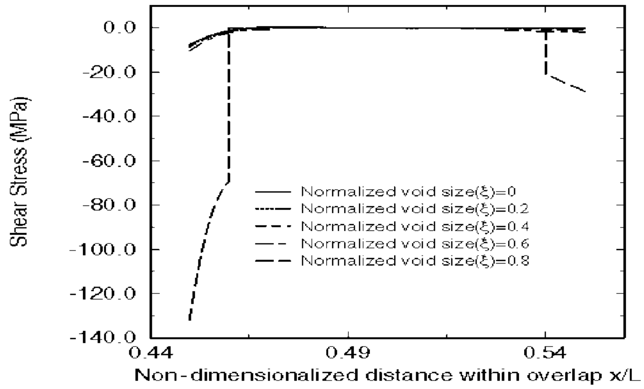

Fig. 10 Shear stress distribution in the bonded joint with various central void sizes. The joint was subjected to a harmonic peeling load of 1 N with the frequency of 185 Hz

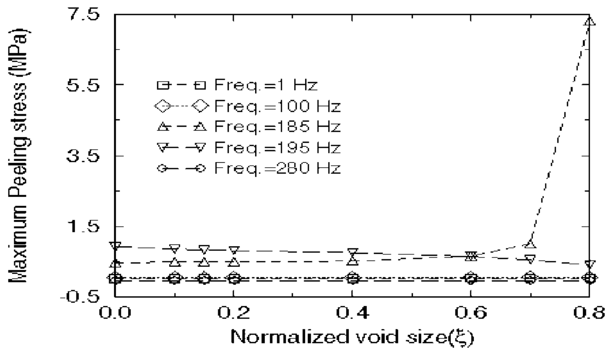

Fig. 11 Maximum peeling stress versus normalized void size, ξ, for bonded joints with the adhesive loss factor of $\eta = 0$. The bonded joints were subjected to a harmonic peeling load of 1 N

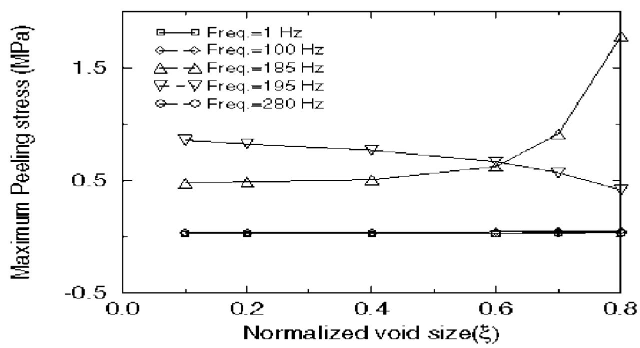

Fig. 12 Maximum peeling stress versus normalized void size, ξ, for bonded joints with the adhesive loss factor of $\eta = 0.5$. The bonded joints were subjected to a harmonic peeling load of 1 N

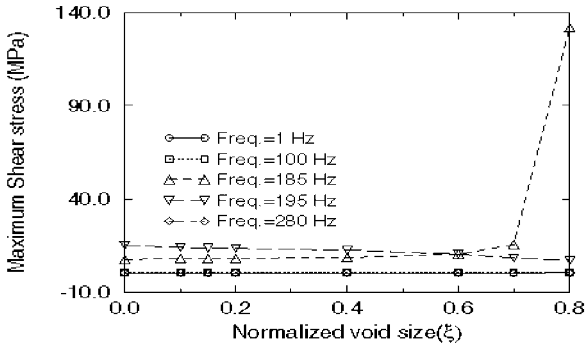

Fig. 13 Maximum shear stress versus normalized void size, ξ, for bonded joints with the adhesive loss factor of $\eta = 0$. The bonded joints were subjected to a harmonic peeling load of 1 N

Figures 11 to 14 also show that the maximum peel and shear stresses in the bonded joint either increase or decrease with the introduction of a central void, with the size exceeding 60 per cent of the overlap length. Maximum peel and shear stresses increase with an increase in the void size when the joint is subjected to the peeling load at the frequency of 185 Hz. In contrast, maximum peel and shear stresses decrease with an increase in the void size when the joint is subjected to a peeling load at the frequency of 195 Hz. These results can be justified considering the changes in the natural frequency with the introduction of a void. A void reduces the natural frequency of the system and brings it closer to 185 Hz and departs further from 195 Hz. Considering the system response (Fig. 3), the amplitude of the system drastically reduces if the frequency of the applied load slightly differs from the natural frequencies of the system.

4 CONCLUSIONS

The dynamic response of an adhesively bonded single-lap joint with a void subjected to a harmonic peeling load is obtained by modelling adherents as Euler–Bernoulli beams. The bond area is modelled as an elastic foundation, which resists both extension and shearing. The adhesive layer is assumed to behave as a viscoelastic material under shear stress and as an elastic material under tension/compression. The numerical results are obtained for aluminium adherents with thicknesses of 3.17 and 12.6 mm and adhesive with various mechanical and damping properties. Based on the analyses, it was found that the first natural frequency of the system corresponded to the natural frequency of the adherent with the thickness of 3.17 mm with one end clamped. The frequency where the peak response occurs was slightly sensitive to the adhesive loss factor. Furthermore, it was found that the natural frequencies were slightly sensitive to the presence of a central void in the bonded area. The results also indicated that the natural frequencies were more sensitive to the void location than its size.

Peel and shear stress distributions in the bond area were obtained for bonded joints with various central void sizes and applying forcing frequencies. The results indicated that a central void covering up to 60 per cent of the overlap might have little effect on maximum peel and shear stresses in the bond area. A central void apparently had a greater effect on the shear stress than the peel stress. Depending on the applied loading frequency, a central void may intensify or may reduce maximum peel and shear stresses in the bond area. This is related to the shift in the natural frequencies of the system. A central void may bring the system natural frequencies closer to the applied loading frequency or may separate it from the applied loading frequency.

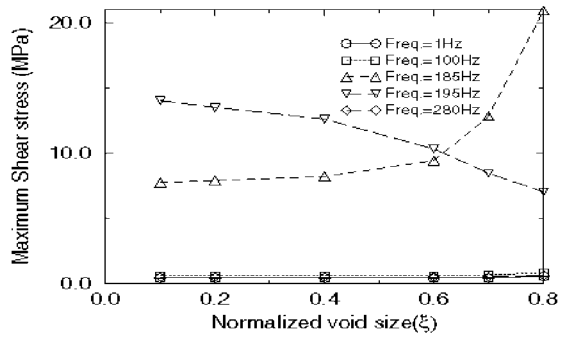

Fig. 14 Maximum shear stress versus normalized void size, ξ, for bonded joints with the adhesive loss factor of $\eta = 0.5$. The bonded joints were subjected to a harmonic peeling load of 1 N

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