Dynamic response of tubular joints with an annular void subjected to a harmonic torsional load

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Abstract: Dynamic responses of adhesively bonded tubular joints subjected to a harmonic torsional load are evaluated. The adherents are assumed to be elastic, and the adhesive is taken to be a linear viscoelastic material. The influence of the adherents and the adhesive properties on the joint response as well as on the shear stress amplitude distribution along the overlap is investigated. Furthermore, the effects of defects such as an annular void in the bond area on the dynamic response and shear stress amplitude distribution in the bond area are studied. The results indicate that, for the tubular joint geometries and properties investigated, the natural frequencies of the joint are little affected by the adhesive loss factor. The natural frequencies of the joint initially increase rapidly with increasing adhesive shear modulus. However, the natural frequencies asymptotically approach a constant value with further increase in adhesive shear modulus. The results further show that the natural frequencies of the joint may not be affected with the presence of a central void in the bond area.

The distribution of the shear stress amplitude in the joint area was obtained. The maximum shear stress was confined to the edge of the overlap for all applied loading frequencies. For the adhesive and adherent properties and geometries investigated, the maximum shear stress amplitude in the joint area was little affected by the presence of a central annular void covering up to 40 per cent of the oveplap length. The result showed that the shear stress amplitude distribution is more sensitive to the void location than to the void size. This was especially pronounced for voids located close to the edge of the overlap.

A central void larger than 40 per cent of the overlap length may be detrimental or beneficial to the joint strength. This depends on the applied loading frequency. A central void reduces the system resonance frequency. This may take the system further away from the applied loading frequency or may bring it closer. A system excited closer to or further from its resonance frequency will develop a higher or lower shear stress amplitude in the bond area.

Keywords: tubular joint with void, harmonic vibration, natural frequency, material damping, adherent properties

NOTATION

G_a	complex shear modulus of the adhesive
G_i	shear modulus of the ith adherent
G_0	real part of the shear modulus of the adhesive
J_i	polar moment of inertia of the ith adherent
l_i	length of the ith section
L	overall length of the bonded joint
R_i	inner or outer radius of the adherents ($i = 1, 2, 3$ and 4)
$T(t)$	applied torque
u_a	axial displacement of the adhesive
u_i	axial displacement of the ith adherent
η	adhesive loss factor
θ_i	rotation of the ith adherent
ρ_a	density of the adhesive
ρ_i	density of the ith adherent
τ	shear stress
ω	angular velocity (rad/s)

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1 INTRODUCTION

The adhesive bonding of components is an attractive method compared with other joining techniques. This is due to ease of manufacturing, cost, weight reduction, reduced thermal exposure and lower stress concentration. Inspite of the many advantages, this method is often applied with caution. The bond strength and adhesive mechanical properties could be severely affected by improper surface preparations and curing procedure and by entrapped void and porosity in the bond area. In order to ensure reliability of these joints, the effects of these defects on both static and dynamic responses of joints must be understood. Over the past 5 years the present authors have studied the effects of defects, voids, adhesive properties and bonded joint geometries and properties on the strength of single-lap joints subjected to axial and peeling loads, tubular joint strength under tension/torsion loading, thermal stress in the bonded joints and non-destructive evaluation of bonded joints [**1–11**]. Both static and dynamic responses of single-lap joints subjected to static and dynamic loadings have been obtained, and the effects of a void in the lap area on the stress distribution and the system resonance frequencies have been investigated. The dynamic response of tubular joints subjected to a harmonic axial load has also been investigated using a shear lag model [**11**]. It has been found that a central void may have little effect on the maximum shear stress in the bond area and axial resonance frequencies. In the present paper, the dynamic response of tubular joints subjected to a harmonic torsional load is investigated.

Earlier research on the static stress analysis of adhesive tubular joints under an axial load was carried out by Lubkin and Reissner [**12**]. They obtained the stress distribution in the lap area, considering both the adherents and the adhesive to be elastic and the adherents to have a thin circular cross-section. Alwar and Nagaraja [**13**] solved the same problem, considering the adhesive to be a viscoelastic material. The shear stress distribution in tubular joints subjected to torsion has been obtained by Adams and Peppiatt [**14**]. Following the same procedure to that reported by Adams and Peppiate, Nayeb-Hashemi *et al.* [**1**] studied the effects of annular voids on the stress distribution in tubular joints subjected to torsion. It was found that for some joints a void may have a negligible effect on the stress distribution in the lap area. Medri [**15**] recently derived the stress distribution in tubular joints subjected to torsion by considering the adhesive to be a viscoelastic material. The stress distribution and dynamic response of tubular joints under other loading conditions such as transverse loading and internal and external loading have been investigated by Rao and Zhou [**16**, **17**], Terekhova and Skoryi [**18**] and Pugno and Surace [**19**]. To the best of the present authors' knowledge, there has been little investigation of the dynamic response of tubular joints subjected to torsion. Furthermore, there has been no systematic study to understand the effects of joint geometries and properties as well as the presence of defects in the bond area on the dynamic response of tubular joints. The present paper will provide further understanding of the effects of tubular joint geometries and properties, as well as the presence of defects such as annular voids, on the dynamic response.

2 THEORETICAL INVESTIGATION

Figure 1 shows a schematic diagram of a tubular joint subjected to harmonic torsional loading. The shear stress in tubular bonded joints subjected to harmonic torsion

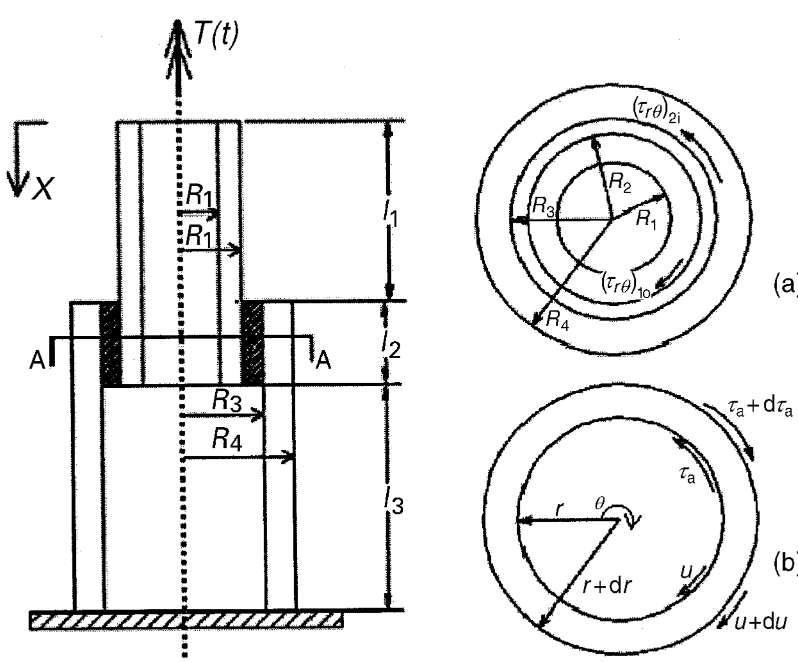

Fig. 1 Schematic diagram of a tubular bonded joint under a harmonic torsional load: (a) applied stresses in the joint cross-section; (b) differential element of adhesive with the applied stresses on its surfaces

is obtained, assuming that the adhesive shears in the circumferential $(r\theta)$ direction and the adherents shear in the axial $(x\theta)$ direction (Fig. 1). The deformation of the adherents in the circumferential $(r\theta)$ direction and the deformation of the adhesive in the axial $(x\theta)$ direction are neglected. The effect of normal stress at the adhesive/adherent interface is also neglected. However, the shear stress can vary across the adhesive thickness. Furthermore, the adherents are considered to be elastic and the adhesive to be linear viscoelastic. The shear modulus, G_a, of the adhesive is assumed to be $G_a = G_0(1 + i\eta)$, where G_0 is the shear modulus, η is the adhesive loss factor and $i = \sqrt{-1}$. The effects of an annular void on the dynamic response and shear stress amplitude distribution are also investigated. Assuming the shear stresses in the inner and the outer surface of adherents 1 and 2 to be $(\tau_{r\theta})_{1o}$ and $(\tau_{r\theta})_{2i}$, and the angular rotations of each adherent at location x to be $\overline{\theta_1}$ and $\overline{\theta_2}$ respectively, the equilibrium equations for an element of each adherent of length dx can be written as

$$\frac{\partial T_1}{\partial x} + (2\pi R_2^2)(\tau_{r\theta})_{1o} - \frac{\pi}{2}(R_2^4 - R_1^4)\rho_1\ddot{\overline{\theta_1}} = 0 \quad (1)$$

$$\frac{\partial T_2}{\partial x} - (2\pi R_3^2)(\tau_{r\theta})_{2i} - \frac{\pi}{2}(R_4^4 - R_3^4)\rho_2\ddot{\overline{\theta_2}} = 0 \quad (2)$$

where ρ_1 and ρ_2 are the densities of adherents 1 and 2, T_1 and T_2 are the applied torques and (R_1, R_2) and (R_3, R_4) are the internal and external radii of adherents 1 and 2 respectively.

The equilibrium equation of an adhesive layer of length dx can be written as

$$2\pi\ [2r\tau_a\,\mathrm{d}r + r^2\,\mathrm{d}_{\tau_a}] - \rho_a J_a\ddot{\overline{\theta_a}} = 0 \quad (3)$$

where τ_a is the shear stress in the adhesive, ρ_a is the adhesive density, $\ddot{\theta}_a$ is the angular acceleration of the adhesive layer and J_a is the polar moment of inertia of the adhesive element. Considering $J_a = 2\pi r^3\,\mathrm{d}r$ and $\overline{\theta_a} = \overline{u_a}/r$, where $\overline{u_a}$ is the adhesive displacement in the θ direction, and assuming that the shear stress in the adhesive layer is $\tau_a = G_a(\partial\overline{u_a}/\partial r)$, equation (3) reduces to

$$2G_a\frac{\partial\overline{u_a}}{\partial r} + r\frac{\partial}{\partial r}\left(G_a\frac{\partial\overline{u_a}}{\partial r}\right) - r\rho_a\overline{u_a} = 0 \quad (4)$$

For an applied harmonic torsional load, the displacements of adherent 1 at its outer surface, adherent 2 at its inner surface and the adhesive layer can be presented as $\overline{u_1}(x, t) = u_1(x)\,e^{iwt}$, $\overline{u_2}(x, t) = u_2(x)\,e^{iwt}$ and $\overline{u_a}(x, r, t) = u_a(x, r)\,e^{iwt}$ respectively. Substituting for the adhesive displacement, equation (4) can be written as

$$\frac{\partial^2 u_a}{\partial r^2} + \frac{2}{r}\frac{\partial u_a}{\partial r} + \beta^2 u_a = 0 \quad (5)$$

where

$$\beta = \sqrt{\frac{\rho_a\omega^2}{G_a}} \quad (6)$$

and ρ_a is the adhesive density. Assuming the displacements of adherents 1 and 2 to be u_1 and u_2 at $r = R_2$ and $r = R_3$ respectively, equation (5) can be solved to obtain the displacement field across the adhesive thickness. The solution can be presented as

$$u_a(r, x) = (u_1\psi_1 + u_2\psi_2)\frac{\cos(\beta r)}{r} + (u_1\psi_3 + u_2\psi_4)\frac{\sin(\beta r)}{r} \quad (7)$$

where

$$\psi_1 = \frac{R_2\sin(\beta R_3)}{z}, \qquad \psi_2 = -\frac{R_3\sin(\beta R_2)}{z},$$
$$\psi_3 = -\frac{R_2\cos(\beta R_3)}{z}, \qquad \psi_4 = \frac{R_3\cos(\beta R_2)}{z}$$

and

$$z = -\sin(\beta R_2)\cos(\beta R_3) + \sin(\beta R_3)\cos(\beta R_2) \quad (8)$$

The shear stress at the adhesive/adherent interface can now be evaluated from equation (7) as

$$(\tau_{r\theta})_{1o} = G_a\frac{\partial u_a}{\partial r}\bigg|_{r=R_2} = G_a[\alpha_1(R_2)u_1 + \alpha_2(R_2)u_2] \quad (9)$$

$$(\tau_{r\theta})_{2i} = G_a\frac{\partial u_a}{\partial r}\bigg|_{r-R_3} = G_a\ [\alpha_1(R_3)u_1 + \alpha_2(R_3)u_2] \quad (10)$$

where $\alpha_1(r)$ and $\alpha_2(r)$ are defined as

$$\alpha_1(r) = \psi_1\frac{-\beta r\sin(\beta r) - \cos(\beta r)}{r^2} + \psi_3\frac{\beta r\cos(\beta r) - \sin(\beta r)}{r^2} \quad (11)$$

$$\alpha_2(r) = \psi_2\frac{-\beta r\sin(\beta r) - \cos(\beta r)}{r^2} + \psi_4\frac{\beta r\cos(\beta r) - \sin(\beta r)}{r^2} \quad (12)$$

The corresponding internal torques in each adherent at any cross-section are written as

$$T_1 = \frac{J_1}{R_2}G_1\frac{\mathrm{d}u_1}{\mathrm{d}x} \quad (13)$$

$$T_2 = \frac{J_2}{R_3}G_2\frac{\mathrm{d}u_2}{\mathrm{d}x} \quad (14)$$

where J_1 and J_2 are the polar moments of inertia of adherents 1 and 2 respectively.

Substituting equations (9), (10), (13) and (14) into equations (1) and (2) yields

$$(R_2^4 - R_1^4)G_1\frac{d^2u_1}{dx^2} + 4\pi G_a R_2^3[\alpha_1(R_2)u_1 + \alpha_2(R_2)u_2] + (R_2^4 - R_1^4)\rho_1\omega^2 u_1 = 0 \quad (15)$$

$$(R_4^4 - R_3^4)G_2\frac{d^2u_2}{dx^2} - 4\pi G_a R_3^3[\alpha_1(R_3)u_1 + \alpha_2(R_3)u_2] + (R_4^4 - R_3^4)\rho_2\omega^2 u_2 = 0 \quad (16)$$

The non-dimensionalized forms of equations (15) and (16) are

$$\frac{d^2u_1}{d\xi^2} + a_1u_1 + a_2u_2 = 0 \quad (17)$$

$$\frac{d^2u_2}{d\xi^2} + a_3u_1 + a_4u_2 = 0 \quad (18)$$

where $\xi = x/L$ and

$$a_1 = \frac{4L^2R_2^3G_a\alpha_1(R_2)}{G_1(R_2^4 - R_1^4)} + \frac{\rho_1\omega^2L^2}{G_1} \quad (19)$$

$$a_2 = \frac{4L^2R_2^3G_a\alpha_2(R_2)}{G_1(R_2^4 - R_1^4)} \quad (20)$$

$$a_3 = -\frac{4L^2R_3^3G_a\alpha_1(R_3)}{G_2(R_4^4 - R_3^4)} \quad (21)$$

$$a_4 = -\frac{4L^2R_3^3G_a\alpha_2(R_3)}{G_2(R_4^4 - R_3^4)} + \frac{\rho_2\omega^2L^2}{G_2} \quad (22)$$

Equations (17) and (18) are valid in the overlap region (l_2). The equilibrium equations for the first and third regions (l_1, l_3) can be written as

$$\frac{d^2u_1}{d\xi^2} + a_5u_1 = 0 \quad (23)$$

$$\frac{d^2u_2}{d\xi^2} + a_6u_2 = 0 \quad (24)$$

where

$$a_5 = \frac{\rho_1\omega^2L^2}{G_1} \quad (25)$$

and

$$a_6 = \frac{\rho_2\omega^2L^2}{G_2} \quad (26)$$

Equations (17) and (18) can be simultaneously solved to obtain the displacement field in the overlap region. This can be written as

$$u_1 = \sum_{j=1}^{4} A_{2j}\, e^{s_{2j}\xi} \quad (27)$$

$$u_2 = \sum_{j=1}^{4} t_jA_{2j}\, e^{s_{2j}\xi} \quad (28)$$

where S_{2j} ($j = 1$ to 4) are the roots of the characteristic equation

$$S^4 + (a_1 + a_4)S^2 + (a_1a_4 - a_2a_3) = 0 \quad (29)$$

and

$$t_j = -\frac{S_{2j}^2 + a_1}{a_2} \quad (30)$$

Similarly, the displacement field in regions (1) and (3) can be written as

$$u_1 = \sum_{j=1}^{2} A_{1j}\, e^{S_{1j}\xi}, \text{ where } S_{11} = i\sqrt{a_5} \text{ and } S_{12} = -i\sqrt{a_5} \quad (31)$$

and

$$u_2 = \sum_{j=1}^{2} A_{3j}\, e^{S_{3j}\xi}, \text{ where } S_{31} = i\sqrt{a_6} \text{ and } S_{32} = -i\sqrt{a_6} \quad (32)$$

The boundary and continuity conditions in the various regions of the overlap are expressed by

$$\frac{du_1}{d\xi} = \frac{2TLR_2}{G_1\pi(R_2^4 - R_1^4)} \quad (33)$$

at $\xi = l_1/L$

$$u_1|_{\text{region 1}} = u_1|_{\text{region 2}} \quad (34)$$

$$\left.\frac{du_1}{d\xi}\right|_{\text{region 1}} = \left.\frac{du_1}{d\xi}\right|_{\text{region 2}} \quad (35)$$

$$\left.\frac{du_2}{d\xi}\right|_{\text{region 2}} = 0 \quad (36)$$

at $\xi = (l_1 + l_2)/L$

$$\left.\frac{du_1}{d\xi}\right|_{\text{region 2}} = 0 \tag{37}$$

$$u_2|_{\text{region 2}} = u_2|_{\text{region 3}} \tag{38}$$

$$\left.\frac{du_2}{d\xi}\right|_{\text{region 2}} = \left.\frac{du_2}{d\xi}\right|_{\text{region 3}} \tag{39}$$

at $\xi = 1$

$$u_2|_{\text{region 3}} = 0 \tag{40}$$

Similar equations are also developed for tubular joints containing an annular void in the overlap area by dividing the overlap into three regions [**20**]. Here, the results will be presented without stating the corresponding equations.

3 RESULTS AND DISCUSSIONS

The results provided here are for the tubular bonded joint configuration shown in Fig. 1. The adherents were bonded using an adhesive with 50 per cent epoxy and 50 per cent hardener, supplied by the Shell Company (Epoxy 828 and

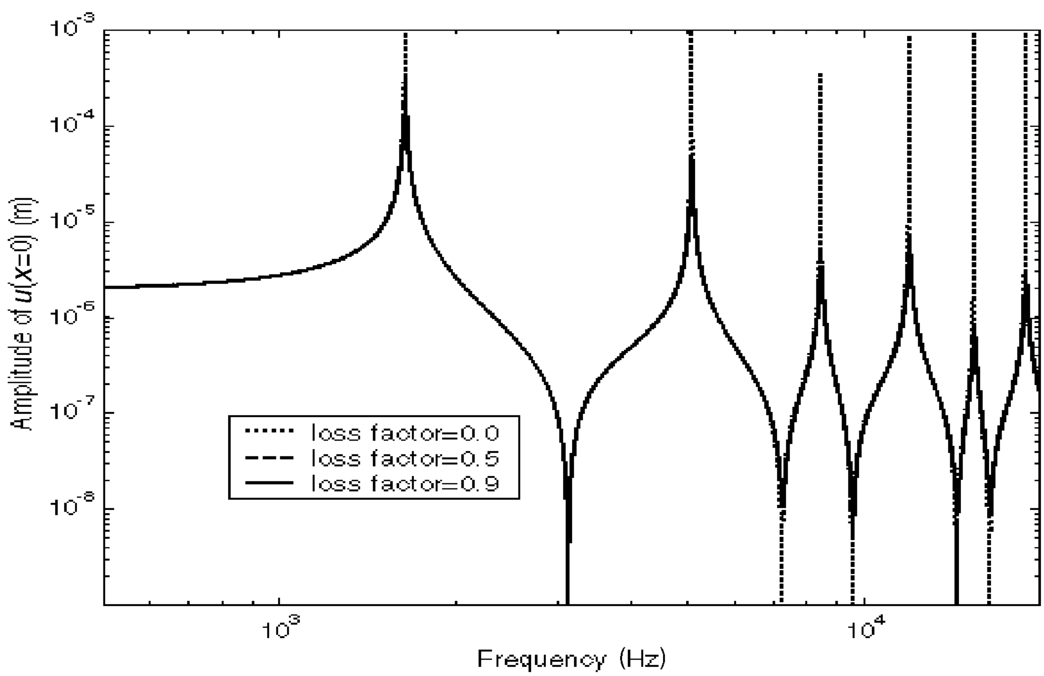

Fig. 2 Effect of the adhesive loss factor, η, of 0, 0.5 and 0.9 on the frequency response of the tubular joint

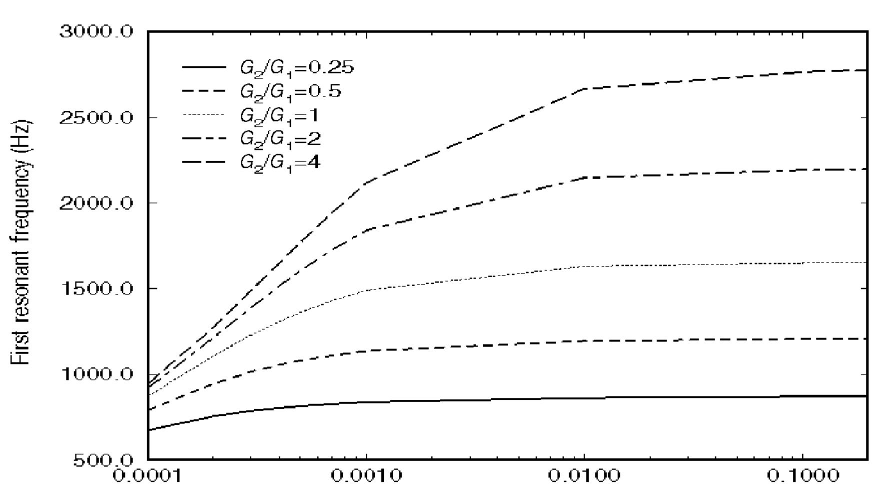

Fig. 3 Effect of adhesive/adherent shear modulus on the first natural frequency of tubular joints (G_1 = constant)

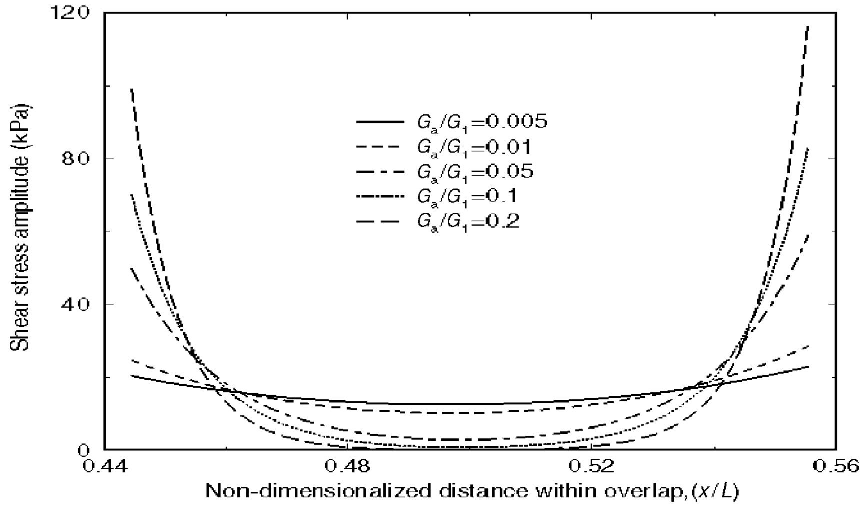

Fig. 4 Distribution of the shear stress amplitude at the adhesive/adherent 1 interface for joints with various G_a/G_1 and adhesive loss factor $\eta = 0$. Tubular joints are subjected to a torsional load of 1 N·m at a frequency of 1000 H_Z

hardener V40). For this mixture of the adhesive, the real part of the shear modulus and its density were 791 MPa and 1200 kg/m^3 respectively. The overlap length was assumed to be 50 mm in all analyses except when it was desired to understand the effect of the overlap length on the dynamic response of the tubular joint. The adhesive was assumed to behave as a linear viscoelastic material with various loss factors, η. For this study, no attempt was made to measure the actual damping value of the adhesive, since it was the effect of adhesive damping on the dynamic response of tubular joints that was of interest. The adhesive damping value is the subject of current ongoing investigations and will be reported upon later. The adherents were 6061-T6 aluminium with an elastic modulus of 69 GPa and a density of 2710 kg/m^3. The inner radius and outer radius of the upper and lower adherents were 0.0 and 17.8 mm and 19.0 and 25.4 mm, respectively. The length of the upper and lower adherents was 250 mm. MATLAB-based codes were written to analyse displacements and related shear stresses.

Figure 2 shows the frequency response of the bonded tubular joint at the point of the applied load. The frequency response was obtained for adhesives with loss factor η ranging from 0 to 1. The results indicate that the adhesive damping affects the amplitude of the vibration and has relatively little effect on the frequencies where peak responses occur, and that, as expected, the amplitude of the system decreases with increase in the adhesive loss factor.

Figure 3 shows the effects of the adhesive and adherent shear modulus on the first resonance frequency of tubular joints. The results indicate that the first resonance frequency of tubular joints increases with increase in the shear modulus of adherent 2 while keeping G_1 constant. The results also show that the first natural frequency of tubular joints

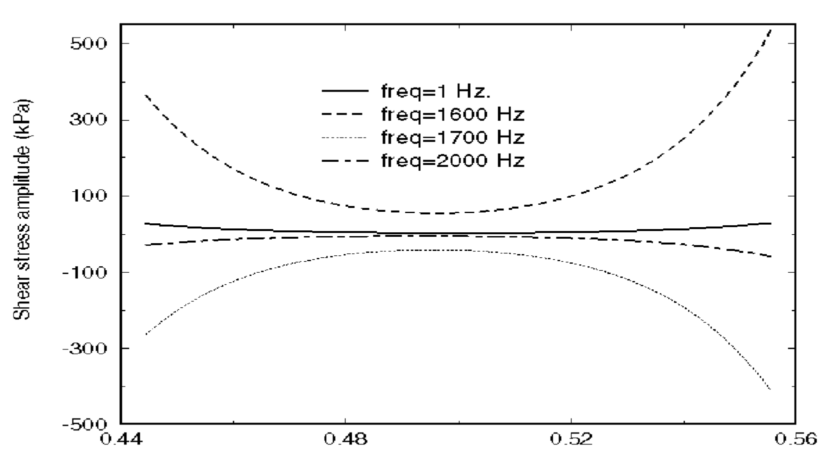

Fig. 5 Distribution of the shear stress amplitude at the adhesive/adherent 1 interface for a joint subjected to a torsional load of 1 N·m at various frequencies. The adhesive is assumed to be elastic, $\eta = 0$

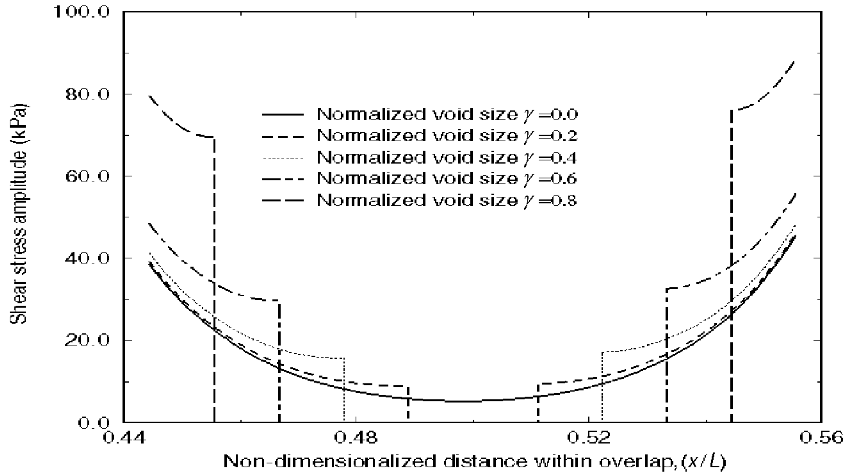

Fig. 6 Shear stress distribution within the overlap section for a tubular joint with adhesive loss factor $\eta = 0$ and with various central void sizes, subjected to a torsional load of 1 N·m at a frequency of 1000 Hz

increases rapidly when using an adhesive with a higher shear modulus. However, beyond a certain shear modulus, the first natural frequency becomes less sensitive to the adhesive shear modulus. These results could be justified considering the shear stress distribution at the adhesive/adherent interface and its variation with the adhesive shear modulus.

Figure 4 shows the distribution of the shear stress amplitude at the adhesive/adherent 1 interface for tubular joints subjected to a harmonic torsional load with an amplitude of 1 N·m at a frequency of 1000 Hz. The results show that the shear stress is distributed fairly uniformly in the overlap for joints with $G_a/G_1 = 0.005$. However, for a higher G_a/G_1, the shear stress distribution is not uniform, with the maximum shear stress amplitude located at the edges of the overlap. The shear stress amplitude distribution is a function of applied loading frequency and increases as the loading frequency approaches the system resonance frequencies (Fig. 5). The shear stress is in phase with the loading frequency when the latter is smaller than the first natural frequency, and it is out of phase when the frequency is greater than the first natural frequency. Furthermore, results indicate that a portion of the overlap may not contribute to the overall resistance of the joint to a torsional load since it has almost zero shear stress and may be considered to be a dead area. The length of this dead zone depends on the relative shear moduli of the adhesive and adherents. In contrast, the natural frequencies of tubular joints depend on the overlap length.

The effect of the size of an annular void and its location on the developed shear stress amplitude in the lap joint is shown in Figs 6 and 7 respectively. Here, void size is defined as γ = void size/overlap length. Results show that the shear stress amplitude distribution within the overlap is

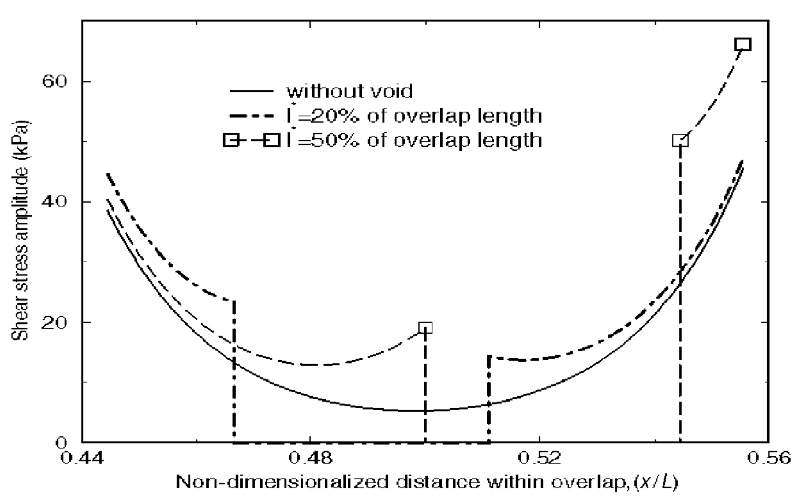

Fig. 7 Shear stress distribution within the overlap section for a tubular joint with adhesive loss factor $\eta = 0$ and with various void locations, subjected to a torsional load of 1 N·m at a frequency of 1000 Hz (l^* is the distance from the edge of the overlap)

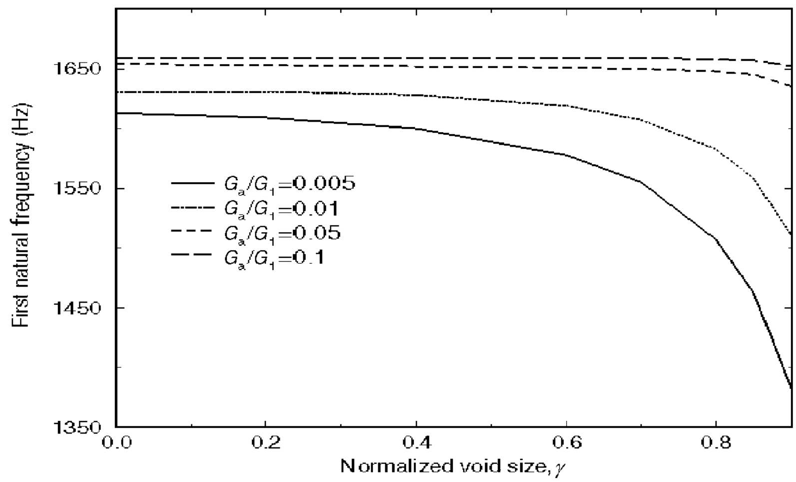

Fig. 8 Effect of annular central void size on the first resonance frequency of a tubular joint with various adhesive/adherent shear moduli

significantly sensitive to both void size and location. A void close to the edges of the overlap significantly intensifies the shear stress amplitude at the edge of the overlap. In contrast, a central void of the same size may have little effect on the shear stress amplitude distribution. This depends on the applied loading frequency.

The results also indicate that, in contrast to the shear stress amplitude, which is extremely sensitive to the void location, the resonance frequencies are less sensitive to the void location. The location of a 30 per cent void was systematically changed from a distance of 10 per cent to a distance of 50 per cent of the overlap length, from the edge of the overlap. The maximum change in the resonance frequencies was 0.2 per cent for the adhesive and adherent properties and geometries investigated.

The effect of central annular void size on the first and second resonance frequencies of the joint is shown in Figs 8 and 9. The results indicate that there may be relatively little change in the resonance frequencies of the joint for joints with a certain central void size. The upper limit of the void size where the resonance frequencies are affected depend on the joint geometries and relative adhesive/adherent shear modulus, G_a/G_1. For example, for joints with $G_a/G_1 > 0.1$, a central void covering up to 90 per cent of the overlap length may have little effect on the joint natural frequencies. However, the resonance frequencies drastically decrease for joints with a larger void size. The results also show that an annular void has more effect on the second natural frequency than on the first resonance frequency. This could be due to the mode shapes and the stress distribution along the overlap for each case.

The reduction in resonance frequencies in the presence of a void may be beneficial or detrimental to the joint strength, depending on the applied loading frequency. For joints

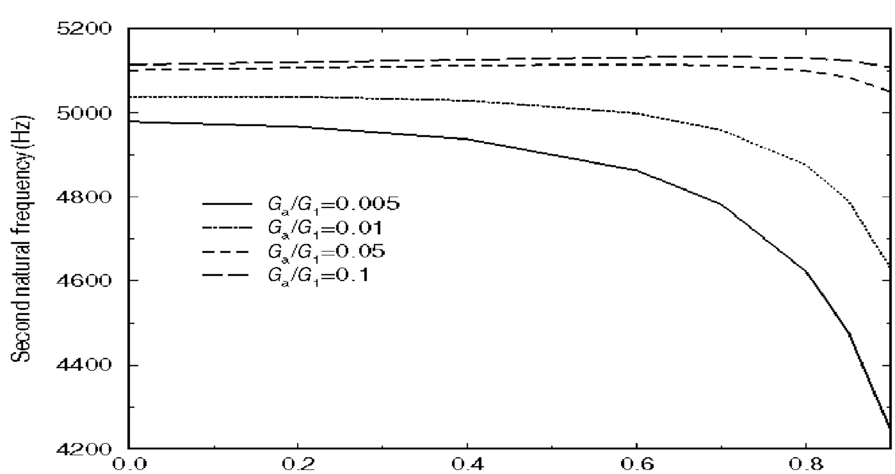

Fig. 9 Effect of annular central void size on the second resonance frequency of a tubular joint with various adhesive/adherents shear moduli

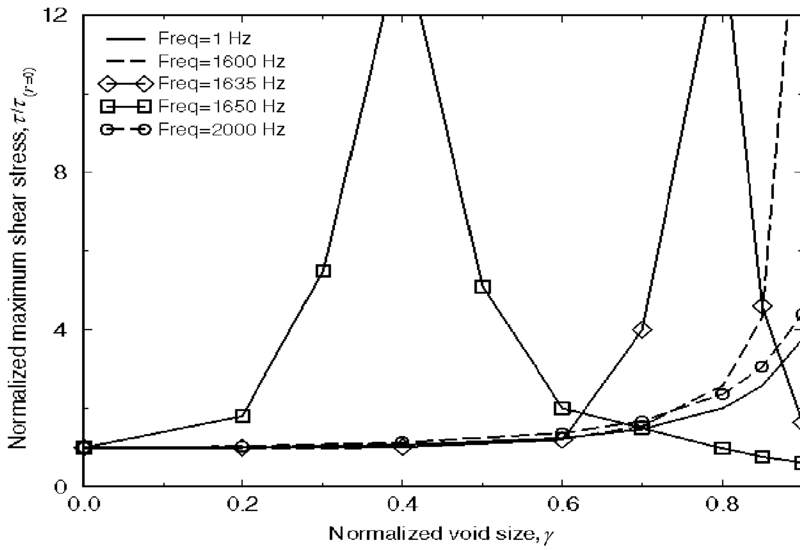

Fig. 10 Maximum shear stress versus normalized void size γ for a tubular joint subjected to a 1 N·m harmonic torque at various frequencies. The adhesive is assumed to be elastic, $\eta = 0$. The first resonance frequency of the joint without the void is 1660 Hz

subjected to a harmonic load at a lower frequency than the natural frequency of the joint without a void, a void may bring the system natural frequency closer to the applied loading frequency, thus increasing the maximum shear stress amplitude in the joint (Fig. 10). However, for joints subjected to a loading frequency greater than the system first natural frequency, a void will take the system further away from the resonance frequency, thus developing a smaller shear stress at the edge of the overlap.

4 CONCLUSIONS

The dynamic response of adhesively bonded tubular joints subjected to a harmonic torsional load has been obtained as a function of the adherent and adhesive mechanical properties. In addition, effects of defects such as an annular void in the overlap area on the system dynamic response have been investigated. The following conclusions can be drawn:

1. The adhesive loss factor has little effect on the frequencies where the peak responses occur. However, as expected, the system response is lower for joints with a higher adhesive loss factor.
2. The system first resonance frequency increases rapidly with increase in G_a/G_1 (adhesive/adherent shear modulus). However, beyond a certain value this increase is less evident.
3. The shear stress distribution in the overlap is obtained for a tubular joint subjected to a harmonic torsional load at several loading frequencies. The maximum shear stress is confined to the edges of the overlap region. Furthermore, for joints with $G_a/G_1 > 0.05$, the middle section of the overlap has almost zero shear stress. This region is termed the dead zone.
4. For the adhesive and adherent properties and geometries studied, the first resonance frequency was little affected by the presence of certain annular central voids in the overlap area. The upper limit of central void size at which the natural frequencies are unaffected depends on the joint geometries and properties. For a larger void size, the system resonance frequencies drastically decrease with increase in void size. The changes in resonance frequencies may prolong the life of the joint or reduce it, depending on the applied loading frequency.
5. An annular void affects the shear stress amplitude distribution along the overlap. Apparently, voids closer to the edges of the overlap have a more significant effect on the shear stress amplitude distribution.

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