+ model
ARTICLE IN PRESS

Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Thin-Walled Structures xx (2006) 1–11

THIN-WALLED STRUCTURES

www.elsevier.com/locate/tws

Buckling of cracked cylindrical thin shells under combined internal pressure and axial compression

A. Vaziri [a,*], H.E. Estekanchi [b]

[a] *Division of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA*
[b] *Department of Civil Engineering, Sharif University of Technology, P.O. Box 11365-9313, Tehran, Iran*

Received 3 October 2005; received in revised form 9 December 2005; accepted 8 February 2006

Abstract

Linear eigenvalue analysis of cracked cylindrical shells under combined internal pressure and axial compression is carried out to study the effect of crack type, size and orientation on the buckling behavior of cylindrical thin shells. Two types of crack are considered; through crack and thumbnail crack. Our calculations indicate that depending on the crack type, length, orientation and the internal pressure, local buckling may precede the global buckling of the cylindrical shell. The internal pressure, in general, increases the buckling load associated with the global buckling mode of the cylindrical shells. In contrast, the effect of internal pressure on buckling loads associated with the local buckling modes of the cylindrical shell depends mainly on the crack orientation. For cylindrical shells with relatively long axial crack, buckling loads associated with local buckling modes of the cylindrical shell reduce drastically on increasing the shell internal pressure. In contrast, the internal pressure has the stabilizing effect against the local buckling for circumferentially cracked cylindrical shells. A critical crack length for each crack orientation and loading condition is defined as the shortest crack causing the local buckling to precede the global buckling of the cylindrical shell. Some insight into the effect of internal pressure on this critical crack length is provided.

Keywords: Cracked cylindrical shell; Buckling behavior; Finite element model

1. Introduction

Shell structures have been widely used in pipelines, aerospace and marine structures, large dams, shell roofs, liquid-retaining structures and cooling towers [1]. Buckling is one of the main failure considerations when designing these structures [2]. Presence of defects, such as cracks, may severely compromise their buckling behavior and jeopardize the structural integrity [3–5]. The post-buckling analysis of cracked plates and shells indicated that the buckling deformation could cause a considerable amplification of the stress intensity around the crack tip [6]. On the other hand, increasing the load can cause propagation of the local buckling leading to the catastrophic failure of the structure [7]. Reviews of the research conducted in the context of buckling behavior of defected plates and shells are presented in references [8–10]. These studies indeed revealed that the sensitivity of the buckling behavior of both plates and shells to the presence of defects highly depends on the loading condition. As an example, in general, the buckling behavior of the cylindrical shells under torsional loading is less sensitive to the presence of a crack than that of a similar axially compressed cylindrical shell [10,11].

The focus of this study is on the bifurcation buckling behavior of cracked cylindrical thin shells under combined internal pressure and axial compression. This is one of the dominant loading conditions for pipelines, liquid-retaining structures and some aerospace and marine structures. The numerical approach seems to be the most promising method, as the analytical formulation of the aforementioned problem is formidably complicated. Even in the case of numerical analysis, the large number of interacting parameters and the complicated shell buckling behavior makes it quite difficult to ascertain generally applicable conclusions [4,10]. The non-linear buckling of thin cylindrical shells with longitudinal cracks subject to above-mentioned loading was studied by Starnes and Rose [12,13]. It was concluded that the non-linear interaction between in-plane stress resultants and the out-of-plane displacements near a crack significantly affects the buckling

* Corresponding author. Tel.: +1 617 496 5167.
E-mail address: avaziri@deas.harvard.edu (A. Vaziri).

0263-8231/$ - see front matter © 2006 Elsevier Ltd. All rights reserved.
doi:10.1016/j.tws.2006.02.004

behavior of the shells. On the other hand, the effect of crack orientation on the buckling behavior of the cracked shells was not studied, which will be shown to have a significant influence on the buckling behavior of these structures.

Here, we have conducted a parametric linear eigenvalue analysis for cracked cylindrical shells under combined internal pressure and axial compression to gain insight into the effect of crack type, size and orientation, as well as the loading condition on the buckling behavior of these structures. Computational models of cracked cylindrical shells with various crack lengths and orientations are developed by employing the meshing scheme proposed by Estekanchi and Vafai [10] for cracked thin plates and shells. The suitability of this meshing scheme for the present study is examined by comparing the numerical results obtained using the proposed meshing scheme with those obtained from finite element models with eight-node quadratic and six-node triangular elements with crack tip singularity. The role of crack orientation, length and type on the buckling behavior of thin cylindrical shells subject to a wide range of combined loading is investigated. Two types of crack are considered; through crack and thumbnail crack.

2. Finite element model and specification of the cylindrical shells

The cylindrical shell is taken to be isotropic and linear elastic material with Young's modulus $E=69$ GPa and Poisson ratio $\nu=0.3$ (corresponding to the elastic properties of aluminum). The computational model of the cylindrical shell has $L=2.0$ m, $R=0.5$ m and $t=0.5$ mm, where L, R and t denote length, radius and thickness of the cylindrical shell, respectively. The present cylindrical shell models has $R/L=0.25$ and $R/t=1000$, categorizing it as a thin shell. In addition, limited number of computations for the model with $R/t=500$ and $R/t=250$ are performed, but the results are not presented in this study for the sake of brevity and since no particular conclusion is made upon them. Maximum length of the crack is taken to be $a/R=0.2$, where a denotes the crack length. The numerical studies are carried out for the crack angle of $\theta=0$–$90°$, measured from the cylinder circumferential line (i.e. $\theta=0°$ corresponds to circumferential direction).

A meshing scheme proposed by Estekanchi and Vafai [10] for cracked thin plates and shells is employed in computational models of cracked cylindrical shells. In this meshing scheme, by approaching the crack tip, the element size reduces incrementally from the constant element size employed in the uncracked region. The validity of this meshing scheme for studying the local behavior of cracked thin plates and shells is established in [10]. One of the main advantages of this meshing scheme is the simplicity it offers for generating the computational models, which is indeed crucial for studies entailing high number of computational models. We developed a computer program to automatically generate finite element models of cracked cylindrical shells based on the proposed mesh pattern. Eight-node shell element is employed in the finite element models. This element has six degrees of freedom at each node and the associated deformation shapes are quadratic in both in-plane directions [14,15]. A mesh sensitivity study is conducted to ensure the independence of the results on the computational mesh. It is concluded that reducing the element size to half of its original size with four levels of zooming at the crack region is capable of capturing the main features of the local buckling behavior of cracked cylindrical shell models. Based on this mesh generation scheme, fourth level of zooming results in crack tip element size of 1/16 of that used in the uncracked region. However, a zoom level of 5 is used in actual analysis for additional accuracy. In the employed meshing scheme, the orientation of the elements is preserved at different levels of mesh zooming, Fig. 1. An alternative approach is based on gradual change of the element orientation at each zoom level. The latter approach is shown to be more effective and accurate for fracture mechanics studies, while the former approach is capable of capturing the main features of local buckling and deformation of cracked cylindrical shells with a remarkable fidelity [10]. The computational models of cylindrical shells with circumferential and angled cracks employed in this study are displayed in Fig. 1. The computations are carried out using ANSYS, a commercial finite element package.

The validity of the approach is examined by comparing numerical results from the proposed meshing scheme with those obtained from models with eight-node quadratic and six-node triangular elements. In latter computational models, the crack tip singularity in the stress–strain field around the crack is incorporated in the models by collapsing the side nodes of the crack tip elements and placing the middle nodes at a quarter distance from the collapsed nodes. Henshell and Shaw [16] showed that the eight-node quadratic plane element generates $1/\sqrt{R}$ and $1/R$ strain singularities by the above-mentioned procedure [17]. In addition, the six-node triangular shell element generates $1/\sqrt{R}$ strain singularities at the crack region by employing an analogous procedure. It can be easily shown that the similar strain singularity can be generated for the eight-node quadratic shell element employed here using the same procedure (ANSYS, Shell 93, [18]), in the range of linear elasticity.

Fig. 2 displays the dependence of the first local buckling load of a circumferentially cracked cylindrical shell on the crack length calculated using three different meshing schemes mentioned above. The calculated compressive buckling load/area, σ_c, is normalized by the theoretical buckling compressive load of the uncracked shell with similar geometry and material properties subject to uniform axial compression, σ_{th}

$$\gamma = \frac{\sigma_c}{\sigma_{th}} \tag{1}$$

where

$$\sigma_{th} = 0.605\frac{Et}{R} \tag{2}$$

and γ denotes the normalized buckling load/area of the cracked cylindrical shells.

+ model
ARTICLE IN PRESS

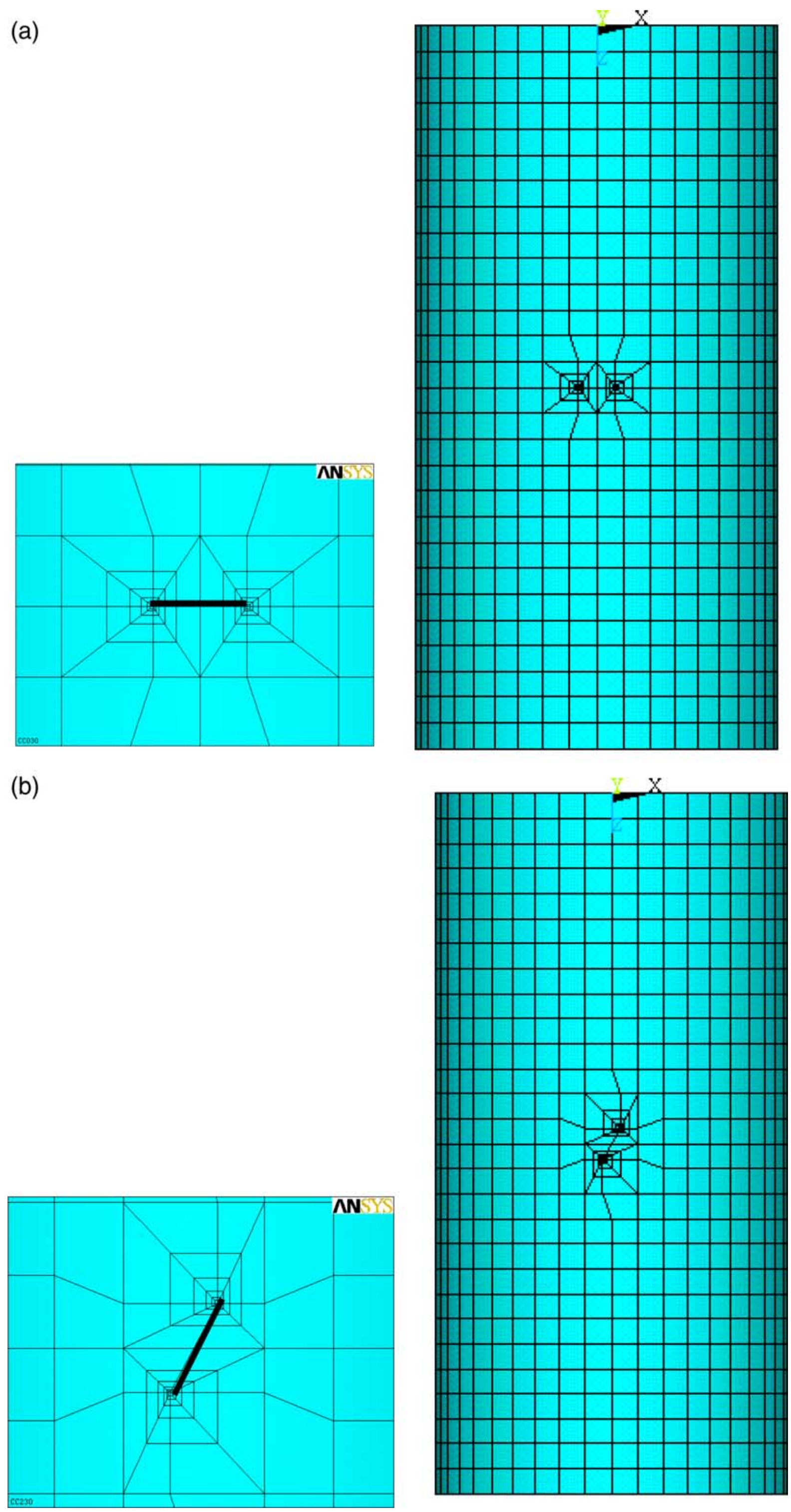

Fig. 1. Computation model of cylindrical shells with (a) a circumferential crack and (b) an angled crack, developed by employing the proposed meshing scheme at the crack region.

The buckling analysis convergence time is significantly higher for models with the crack tip singularity. In addition, introducing the internal pressure in models with crack tip singularity may cause the computations to diverge. No difference is revealed between the first local buckling shapes of circumferentially cracked shells obtained using each of the above-mentioned meshing schemes. It is noteworthy that the proposed meshing scheme replicates the results of the models with crack tip singularity more closely for cylindrical shells with axial and angled cracks. The results of this comparative study indicate that the employed mesh pattern is capable of capturing the local buckling behavior of cracked cylindrical

+ model ARTICLE IN PRESS

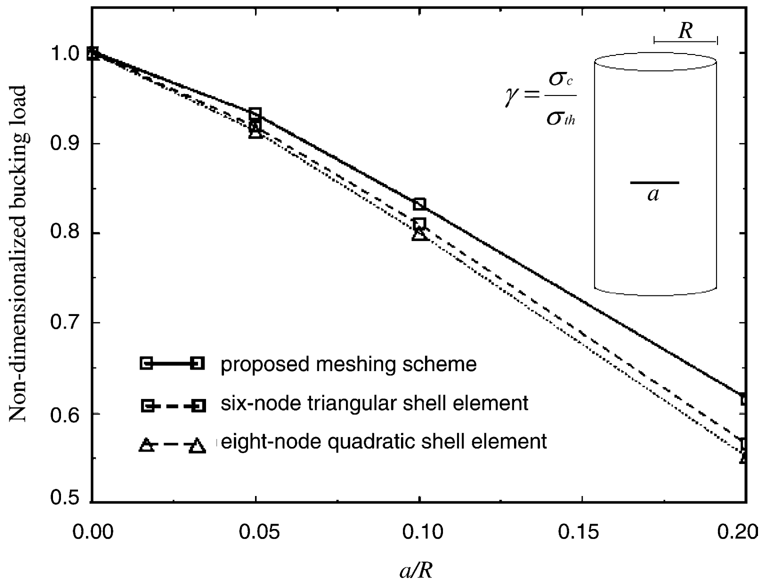

Fig. 2. Non-dimensionalized first buckling load of circumferentially cracked cylindrical shells, γ, versus the normalized crack length, obtained using different meshing schemes. Models that employ eight-node and six-node shell elements have the crack tip singularity in the stress–strain field around the crack as explained in Section 2.

shells with a remarkable fidelity, while it is very effective computationally. The proposed meshing scheme is used to generate the finite element models of the cylindrical shells with various crack lengths and orientations. We also studied the buckling behavior of composite cracked cylindrical shells subjected to axial compression using this meshing scheme [19]. The results suggested that the ply sequence of the composite can be designed to minimize the sensitivity of cylindrical shell buckling behavior to the presence of defects [19].

Two crack models are considered in this study; through crack and thumbnail (part through) crack. Through cracks are modeled by allowing the relative displacement and rotation (in all six degrees of freedom) of the neighbor nodes located at two edges of the crack. The thumbnail crack model is assessed by constraining the relative linear displacement of the neighbor nodes at two edges of the crack, while no constrain is imposed in regard to their relative rotational displacement. This model mimics a plastic hinge with negligible bending resistance at the crack edge.

3. Buckling behavior of cracked cylindrical shells

The results presented in this section are based on bifurcation buckling analysis of cracked cylindrical shells under combined compression and internal pressure. A normalized loading parameter, λ, is defined as the ratio of the induced constant hoop stress due to the internal pressure in an uncracked cylindrical shell, σ_{hoop}, to the uniform axial compressive stress, σ_{axial}

$$\lambda = \frac{\sigma_{\text{hoop}}}{\sigma_{\text{axial}}} \tag{3}$$

where

$$\sigma_{\text{hoop}} = \frac{pR}{t} \tag{4}$$

where p denotes the internal pressure applied to the cylindrical shell.

The first and second buckling loads of the uncracked shell slightly increase on increasing the internal pressure as shown in Fig. 3. These results are consistent with the well-known classical results for non-cracked cylindrical shells under combined loading [20]. It should be noted that in all the analyses in this paper, σ_{axial} is the net resultant axial stress, i.e. the axial stress resulting from external loading and internal pressure. The buckling shapes of uncracked cylindrical shells are referred as global buckling shapes throughout the paper. For the shell geometries and properties studied here, the presence of a crack does not have a significant effect on the global buckling shapes of the shell. A cracked shell could buckle locally (at the crack region). Sections 3.1 and 3.2 present the results of the numerical analyses for the buckling behavior of cracked cylindrical shells with axial and circumferential cracks, respectively. The role of crack orientation on the buckling behavior of cracked shells is discussed in Section 3.3. The local buckling shapes and the associated buckling loads are not sensitive to the crack location (as long as the effect of shell boundary is negligible). All the results presented here are for cracks located at the middle of the cylindrical shell, Fig. 1.

It is noteworthy that bifurcation buckling analysis does not provide any information regarding the post-buckling behavior of the structure and also does not account for the effect of pre-buckling deformations. However, most of the practical design formulas are usually based on the bifurcation buckling analysis, which leads to a proper approximation to the real

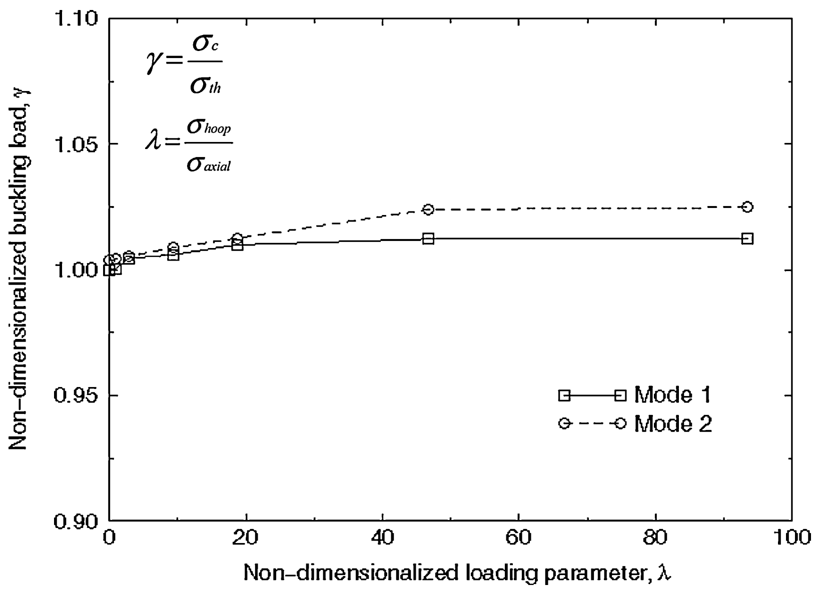

Fig. 3. Non-dimensionalized first and second buckling loads of the uncracked cylindrical shell, γ, versus non-dimensionalized loading parameter, λ.

buckling load in most practical cases [1]. Therefore, in most of the cases, linear eigenvalue analysis may be sufficient for the preliminary design evaluation, but if there is a concern about material non-linearity or unstable post-buckling response, a non-linear post-buckling analysis should also be performed [21].

3.1. Buckling behavior of cylindrical shells with an axial crack

The computational model is used to study the first two buckling modes of axially cracked shells under combined internal pressure and axial compression. Local buckling can precede the global buckling of axially cracked cylindrical shells depending on the crack length and the internal pressure. In general, buckling loads associated with local buckling of the cylindrical shell decrease on increasing the crack size, as expected. The effect of internal pressure, however, is the result of interplay of two mechanisms: (a) the stiffening effect of radial tension that tends to increase the buckling load of perfect cylindrical shells or those with minor geometric imperfections, and (b) the local disturbance of the stress field, in combination with the induced local compressive stress at the crack edges tend to facilitate the local buckling. Fig. 4(a) presents the dependence of first and second normalized buckling loads of the axially cracked cylindrical shell on the normalized loading parameter λ, for two different crack lengths, $a/R=0.05$ and $a/R=0.15$. The first local buckling mode of the cylindrical shell with an axial crack is displayed in Fig. 4(b). For the crack length of $a/R=0.05$, the buckling load associated with the first buckling mode increases on increasing the internal pressure. This can be attributed to the stiffening effect resulted from hoop stresses as in the case of cylinders with small imperfections [20]. Meanwhile, the buckling load associated with the second buckling mode decreases till the limit that its associated buckling shape precedes and replaces the first buckling mode of the cylindrical shell. Further increase in the internal pressure, $\lambda > 4.5$, results in both first and second buckling loads to decrease on increasing the internal pressure. For the crack length of $a/R=0.15$, both first and second buckling shapes correspond to the local buckling modes of the cylindrical shell. In this case, the weakening effect of internal pressure is the dominant mechanism and the buckling load falls rather sharply on increasing the internal pressure (even at low internal pressures). Increasing the internal pressure results in a significant reduction of the buckling load in the low-pressure region. The effect of the internal pressure on the buckling load of the cylindrical shell with an axial crack is depicted in Fig. 5 for various crack lengths. The buckling loads associated with local buckling mode of cylindrical shell are significantly more sensitive to the internal pressure for cylindrical shells with longer axial cracks. The results further indicated that the internal pressure does not affect the local buckling mode shapes significantly. The buckling mode shape resembles that of Fig. 4(b) with minor changes in relative curvatures. It can be seen that at low internal pressure, internal pressure tends to increase the buckling load for a/R ratios lower than approximately 0.07. In the high-pressure region, the internal pressure has a negative effect on buckling loads for all crack lengths studied here.

3.2. Buckling behavior of cylindrical shells with a circumferential crack

Fig. 6 shows the first and second normalized buckling loads of circumferentially cracked cylindrical shells versus the normalized loading parameter, λ, for various crack lengths. The results are presented for both through crack and thumbnail (part through) crack. Similar to the results presented in Section 3.1, presence of a very short crack does not alter the buckling shapes and the associated buckling loads considerably. By

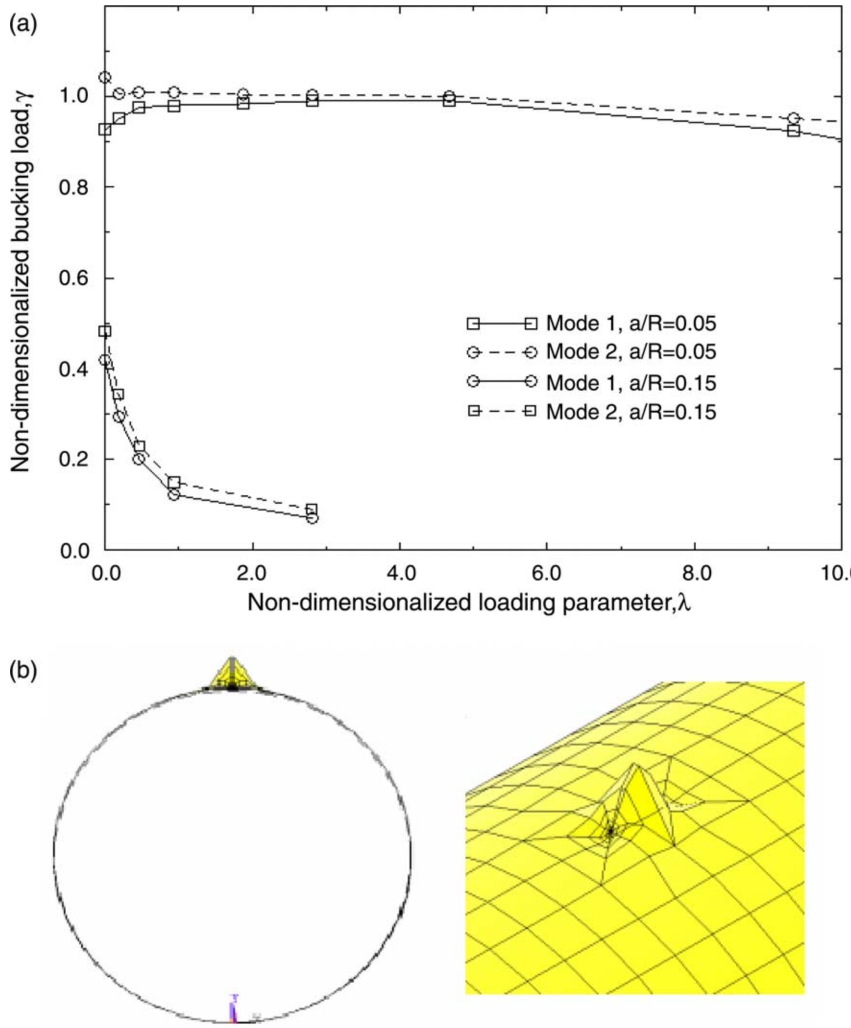

Fig. 4. (a) Non-dimensionalized first and second buckling loads of axially cracked cylindrical shells, γ, versus non-dimensionalized loading parameter, λ. The crack is a through crack with two different normalized lengths, $a/R = 0.05$ and 0.15. (b) The first local buckling shape of the cracked cylindrical shell with an axial crack of $a/R = 0.15$. This buckling shape is not considerably sensitive to the internal pressure.

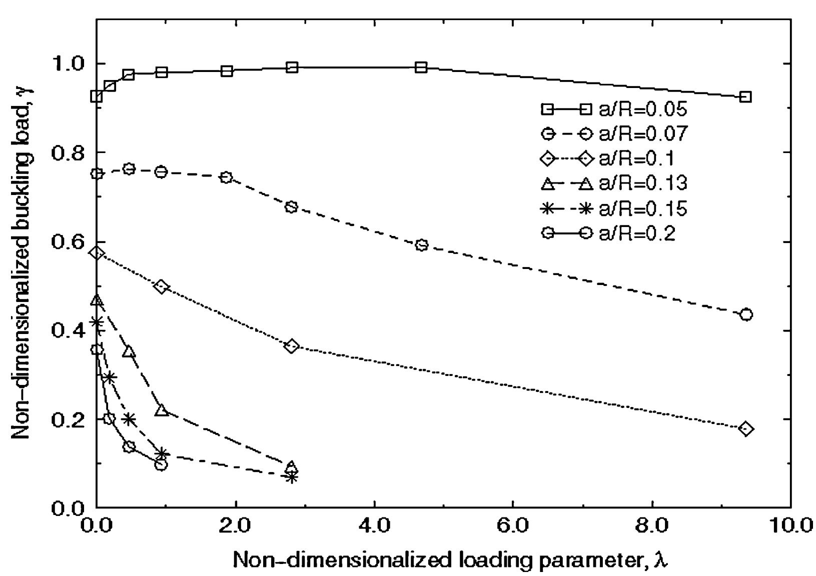

Fig. 5. Non-dimensionalized first buckling load of axially cracked cylindrical shells, γ, versus non-dimensionalized loading parameter, λ, for different normalized crack lengths, a/R. The crack is a through crack.

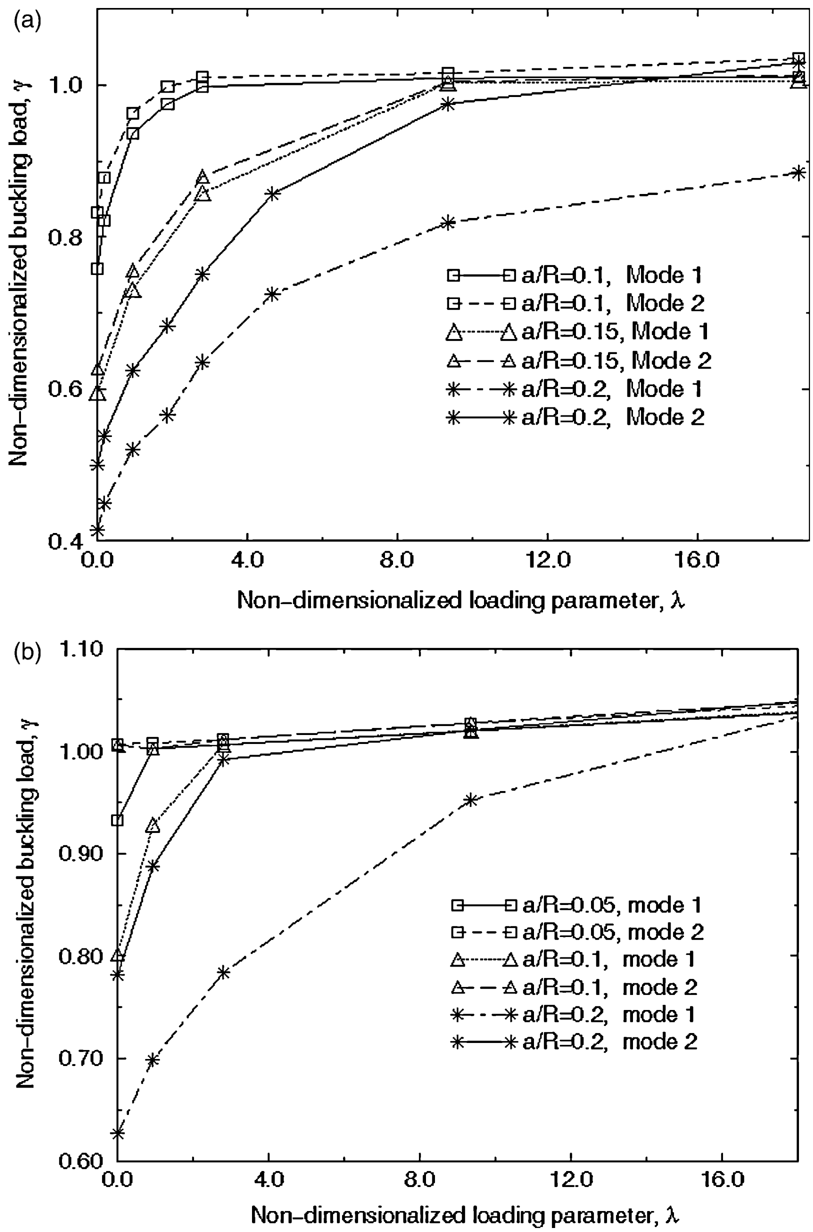

Fig. 6. Non-dimensionalized first and second buckling load of cylindrical shells with a circumferential crack, γ, versus non-dimensionalized loading parameter, λ, for various normalized crack lengths, a/R for computational models with (a) through crack (b) thumbnail crack.

increasing the crack length, local buckling precedes the first global buckling mode. For the cylindrical shell with circumferential crack, the internal pressure stabilizes the cylindrical shell by suppressing the local buckling at the crack region. This stabilizing effect can be explained by considering the effect of internal pressure on the first local buckling mode of the cylindrical shell with a circumferential crack. Fig. 7 displays the first buckling mode of the cylindrical shell with a circumferential crack of $a/R=0.2$, for two loading parameters, $\lambda=0$ and 10. Study of mode shapes with various loading parameters reveals that the internal pressure stabilizes the shell against buckling by suppressing the first possible local buckling shape and causing higher local buckling shapes to become critical. For instance in Fig. 7, the first buckling mode of the circumferentially cracked shell for $\lambda=10$ is similar to the second local buckling mode of the same shell with no internal pressure, $\lambda=0$. The internal pressure stabilizes the shell against both global and local buckling. This is in contrast to the case of axially cracked shell, where the global buckling precedes the local buckling at high internal pressures. The internal pressure has a more profound role on enhancing the resistance of cylindrical shells against local buckling in comparison to global buckling, especially for the case of cylindrical shells with a through crack. If no internal pressure is applied, buckling loads associated with the local buckling modes of the cylindrical shell with a through circumferential crack are significantly lower than the buckling loads of the same shell with a circumferential thumbnail crack of equal length. The

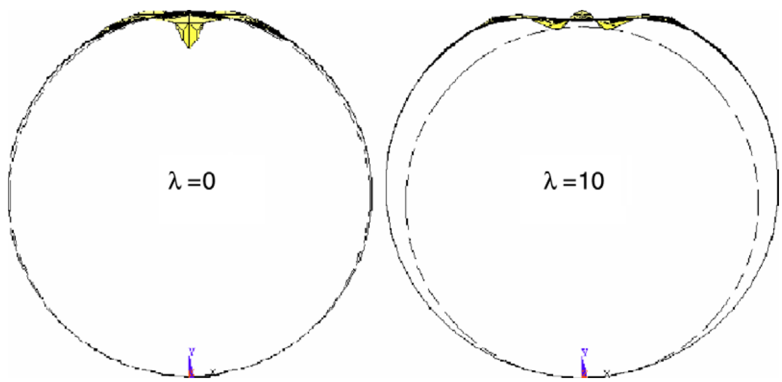

Fig. 7. First local buckling modes of cylindrical shells with a circumferential through crack with no internal pressure ($\lambda=0$) and subject to high internal pressure ($\lambda=10$), where dashed and solid lines show the unloaded cylindrical shell and its first buckling mode under combined internal pressure and axial compression, respectively.

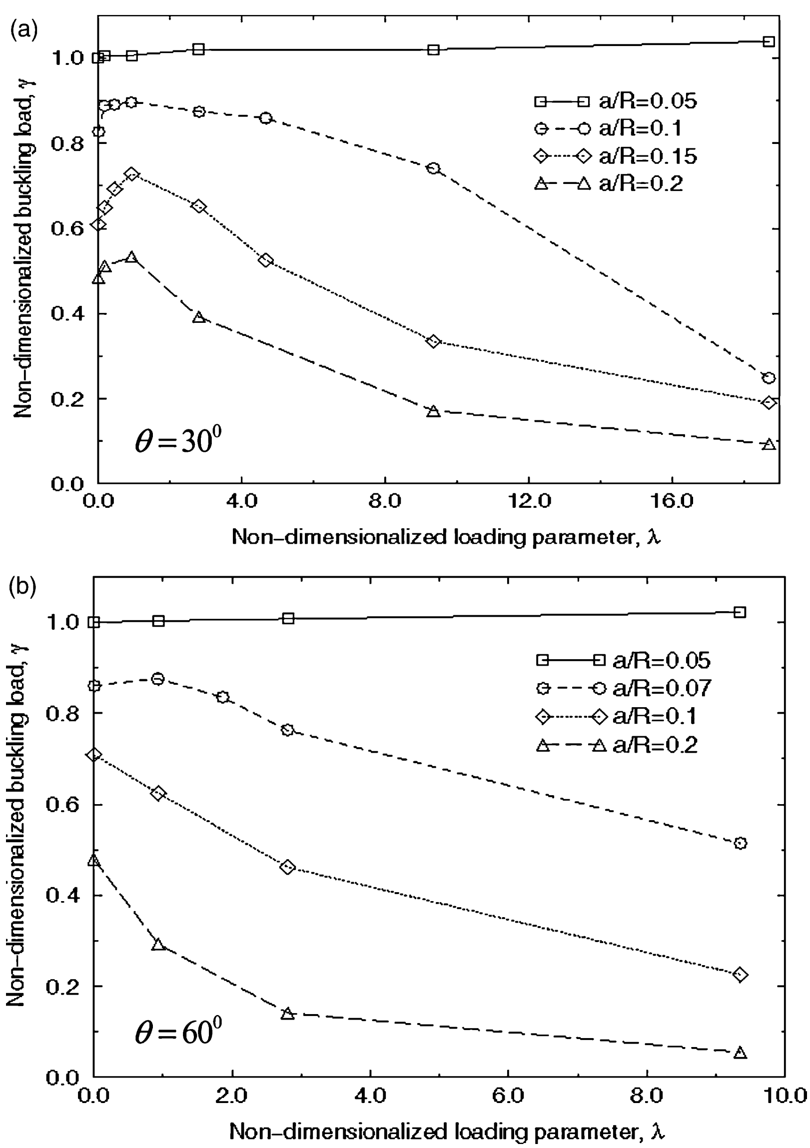

Fig. 8. Non-dimensionalized first buckling load of cylindrical shells with an inclined through crack, γ, versus non-dimensionalized loading parameter, λ, for different crack lengths with crack orientations of (a) $\theta=30°$ and (b) $\theta=60°$.

sensitivity of buckling loads associated with local buckling of the cylindrical shell to the crack type vanishes on increasing the shell internal pressure. It is noteworthy that the sensitivity of the buckling loads of the cylindrical shell to the internal pressure depends on the crack length. For example, for the crack length, $a/R = 0.2$, the first and second buckling modes of the cylindrical shell with a through circumferential crack are local buckling modes in the loading range studied here, although buckling loads become significantly higher in the presence of internal pressure.

3.3. The effect of crack orientation on the buckling behavior of cracked cylindrical shells

In this section, the effect of crack orientation on the buckling behavior of cracked cylindrical shells under combined internal pressure and axial compression is studied by emphasizing on two crack orientations of 30° and 60°, measured from the circumferential line of the cylindrical shell. The first normalized buckling load of cracked shells with crack angle of 30° and 60° versus the normalized loading parameter are depicted in Fig. 8(a) and (b), respectively. The effect of internal pressure on the local buckling of cylindrical shells is the result of interplay of two mechanisms: (1) the local disturbance of the stress field, in combination with the induced local compressive stress at the crack edges, which tends to decrease the local buckling loads (the dominant effect for axially cracked shells); (2) the stabilizing effect of the internal pressure, which tends to suppress the lower buckling mode of cylindrical shells (the dominant effect for circumferentially cracked shells). The relative influence of these two mechanisms on the local buckling behavior of cracked cylindrical shells depends on the crack orientation and the internal pressure. For a cracked cylindrical shell with a crack oriented at 30° under relatively low pressure, $\lambda < 2$, the former mechanism associated with the internal pressure dominates the buckling behavior of the

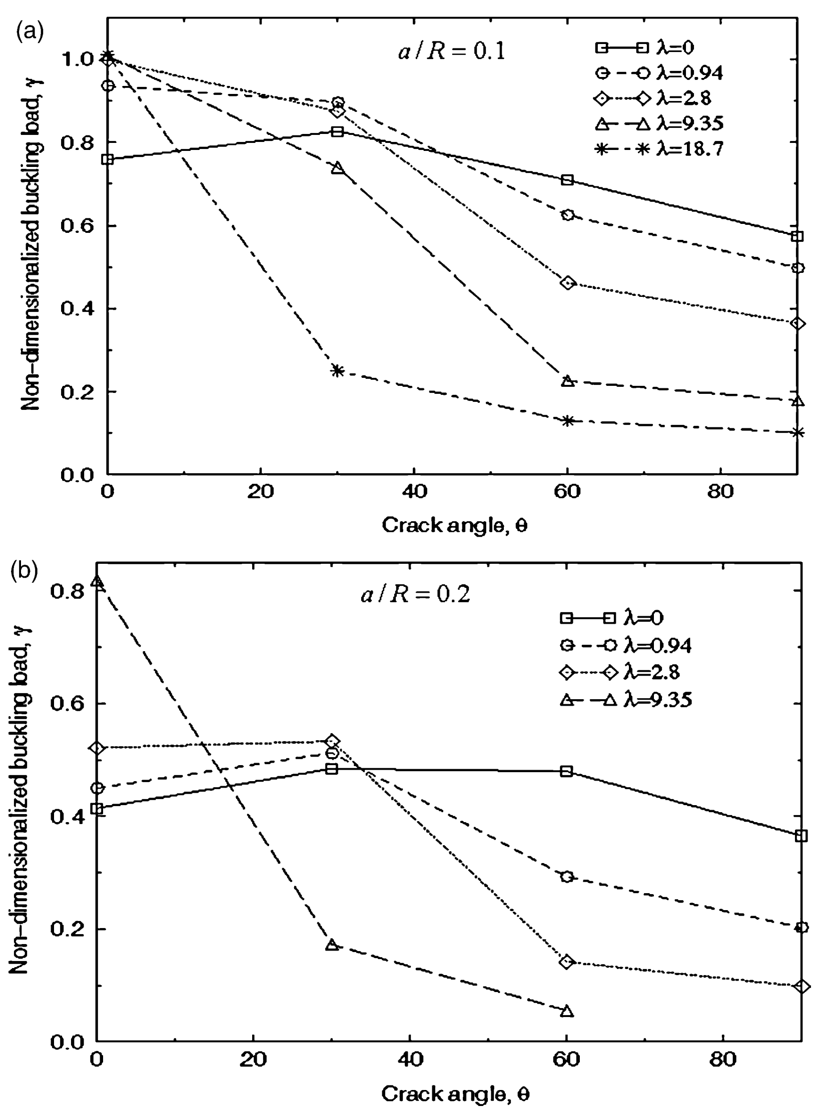

Fig. 9. Non-dimensionalized first buckling load of cylindrical shells with an inclined through crack, γ, versus the crack angle for various non-dimensionalized loading parameter, λ, and normalized crack lengths of (a) $a/R = 0.1$ and (b) $a/R = 0.2$.

shell. By increasing the internal pressure, the buckling load associated with the first local buckling mode of the cylindrical shell reduces considerably, Fig. 8(a). For a crack oriented at 60° from the circumferential line, the second mechanism dominates the local buckling of the cylindrical shells with $a/R > 0.1$ and the internal pressure reduces the buckling load associated with the first buckling mode of the cracked cylindrical shell. Fig. 9(a) and (b) shows the effect of the crack orientation on the first buckling load of the cracked cylindrical shell for two normalized crack lengths, $a/R = 0.1$ and 0.2, respectively. If no internal pressure is applied, the buckling loads associated with the local buckling of the cracked cylindrical shell are not sensitive to the crack orientation. For cylindrical shells subject to relatively low internal pressure, the buckling load associated with the first buckling mode of the cylindrical shell increases by increasing the crack angle from the circumferential orientation. However, further increase in the crack angle reduces the first buckling load of the shell. In addition, presence of an axial crack results in the maximum reduction of the buckling load associated with the first local buckling shape of the cylindrical shell for all crack orientations.

Similar to axially and circumferentially cracked shells, for a crack shorter than a critical length $(a/R)_{\text{Critical}}$, the local buckling does not precede the global buckling of the cylindrical shell. The role of crack orientation and internal pressure on this critical crack length are shown in Fig. 10. For cylindrical shells with no internal pressure, the critical crack length is not considerably sensitive to the crack orientation. However, the internal pressure increases the sensitivity of the critical crack length to the crack orientation. For an axially cracked cylindrical shell, increasing the internal pressure decreases the critical length. In contrast, the internal pressure increases the critical crack length for a circumferentially cracked shell. For a cracked cylindrical shell with a crack oriented at 30°, an initial increase of the internal pressure causes the critical length to decrease, while further increase of the internal pressure increases the critical length. The effect of crack type on the critical length is also investigated. It is observed that for circumferentially cracked shells, crack type has no significant effect on the critical length. In contrast, for the case of axially cracked shells, this critical length is considerably larger for shells with a thumbnail crack than shells with a through cracked.

4. Concluding remarks

Buckling analyses are performed to investigate the bifurcation buckling behavior of perfect and cracked cylindrical shells under combined internal pressure and axial compression. The internal pressure does not affect the overall buckling behavior of perfect cylindrical shells in the range of internal pressure studied here. Similar results are obtained regarding the effect of internal pressure on the resonant frequencies of cylindrical shells [22]. On the other hand, presence of a crack may significantly alter the buckling behavior of cylindrical shells by provoking local buckling as the dominant buckling mode of the cylindrical shell. The local buckling may precede the global buckling of cylindrical shells for crack longer than a critical length. This critical length depends on the crack orientation and the loading condition. The results presented in [22] indicates that the length of a crack, which leads to a considerable change in resonant frequencies of cracked cylindrical shells is significantly longer than the critical crack length associated with the buckling behavior. The internal pressure may stabilize the shell against local buckling by suppressing the lower modes associated with local buckling or may provoke the local buckling of cylindrical shells due to

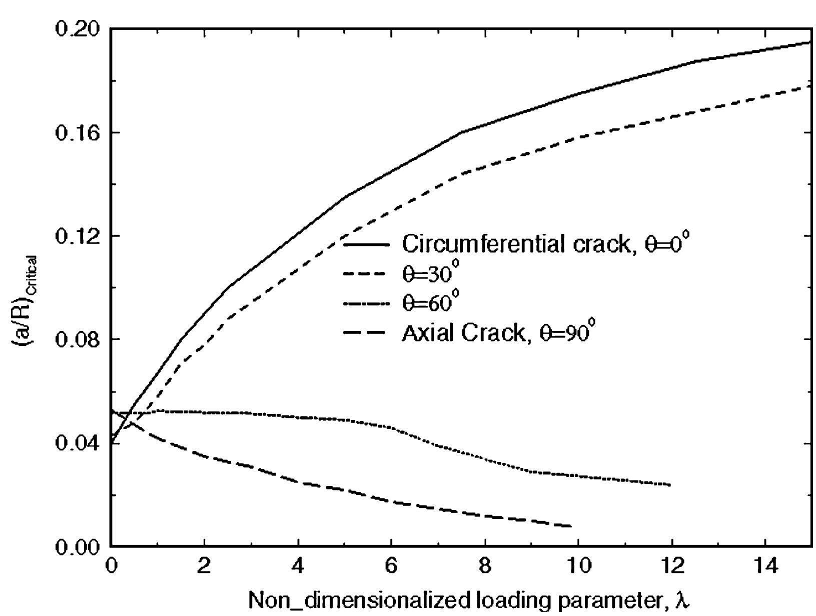

Fig. 10. Critical crack length of the cylindrical shell with a through crack versus non-dimensionalized loading parameter, λ, for different crack orientations.

stress concentration. This mainly depends on the shell loading parameter and the crack orientation.

Acknowledgements

The authors would like to acknowledge Professors Hamid Nayeb-Hashemi (Department of Mechanical, Industrial and Manufacturing Engineering, Northeastern University) and John W. Hutchinson (Division of Engineering and Applied Sciences, Harvard University) for many insightful discussions.

References

[1] Farshad M. Design and analysis of shell structures. Dordrecht: Kluwer; 1992.
[2] Buckling of circular cylindrical shells under axial compression. In: Budiansky B, Hutchinson JW, editors. Contributions to the theory of aircraft structures. Netherlands: Delft University Press; 1972. p. 239–60.
[3] Hutchinson JW, Tennyson RC, Muggeridge DB. Effect of local axisymmetric imperfection on the buckling of a cylindrical shell under axial compression. AIAA J 1972;9:48–52.
[4] El Naschie MS. Branching solution for local buckling of a circumferentially cracked cylindrical-shell. Int J Mech Sci 1974;16:689–97.
[5] Barut A, Madenci A, Britt VO, Starnes JH. Buckling of a thin, tension-loaded, composite plate with an inclined crack. Eng Fract Mech 1997;58:233–48.
[6] Riks A, Rankin CC, Brogan FA. The buckling of a central crack in a plate under tension. Eng Fract Mech 1992;26:1023–42.
[7] Chater E, Hutchinson JW. On the propagation of bulges and buckles. J Appl Mech 1984;51:1–9.
[8] Khamlichi A, Bezzazi M, Limam A. Buckling of elastic cylindrical shells considering the effect of localized axisymmetric imperfections. Thin Wall Struct 2004;42(7):1035–47.
[9] Hilburger MW, Starnes JH. Effects of imperfections of the buckling response of composite shells. Thin Wall Struct 2004;42(3):369–97.
[10] Estekanchi HE, Vafai A. On the buckling of cylindrical shells with through cracks under axial load. Thin Wall Struct 1999;35(4):255–74.
[11] Estekanchi HE, Vafai A, Kheradmandnia K. Finite element buckling analysis of cracked cylindrical shells under torsion. Asian J Civ Eng 2002;3(2):73–84.
[12] Starnes JH, Rose ChA. A nonlinear response of thin cylindrical shells with longitudinal cracks and subjected to internal pressure and axial compression loads. Proceedings of the 1997 38th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference; 1997. p. 2213–23 [Part 3 (of 4)].
[13] Starnes JH, Rose ChA. Buckling and stable tearing responses of unstiffened aluminum shells with long cracks. AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics and materials conference. Proceedings of the 1998 39th AIAA/ASME/ASCE/AHS/ASC structures, structural dynamics, and materials conference and exhibit and AIAA/ASME/AHS adaptive structures forum; 1998. p. 2389–402 [Part 3 (of 4)].
[14] Ahmad S, Irons BM, Zienkiewicz OC. Analysis of thick and thin shell structures by curved finite elements. Int J Numer Methods Eng 1970;2(3):419–51.
[15] Cook RD. Concepts and applications of finite element analysis. 2nd ed. New York: Wiley; 1981.
[16] Henshell RD, Shaw KG. Crack tip finite elements are unnecessary. Int J Numer Methods Eng 1975;9:495–507.
[17] Bathe KJ. Finite element procedures. Englewood Cliffs, NJ: Prentice-Hall; 1996.
[18] ANSYS 6.1, Manual, Element References.
[19] Vaziri A, Nayeb-Hashemi H, Estekanchi HE. Buckling of the composite cracked cylindrical shells subjected to axial load. Proceedings of the ASME congress and exposition; 2003.
[20] Bushnell D. Computerized buckling analysis of Shells. Dordrecht: Martinus NijHoff; 1985.
[21] Hutchinson JW. Post-bifurcation behavior in the plastic range. J Mech Phys Solids 1973;21(3):163–90.
[22] Vaziri A, Nayeb-Hashemi H, Estekanchi HE. Dynamic response of cracked cylindrical shells with internal pressure. Proceedings of the ASME congress and exposition; 2002.